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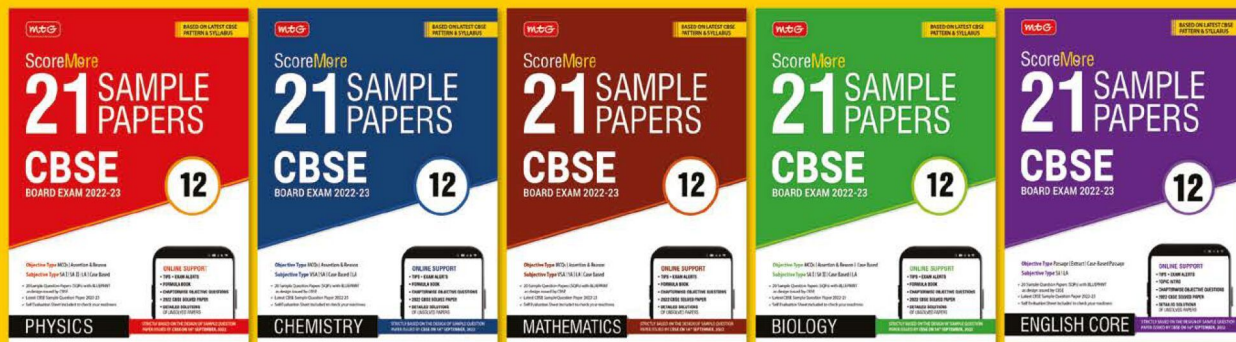
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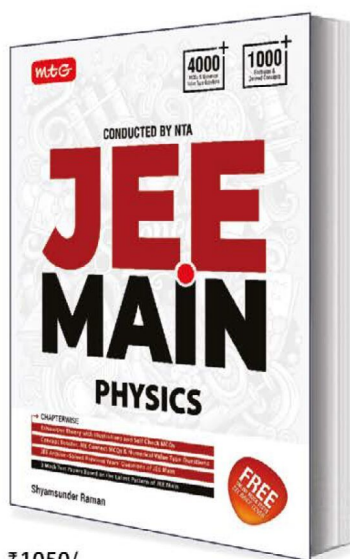
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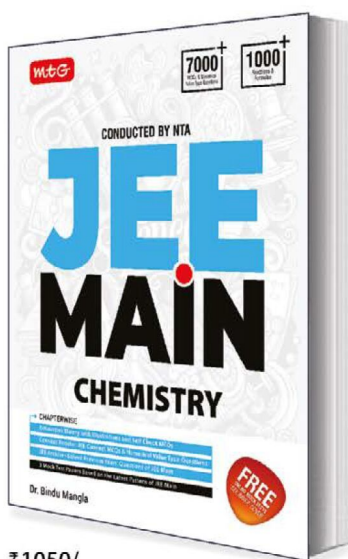
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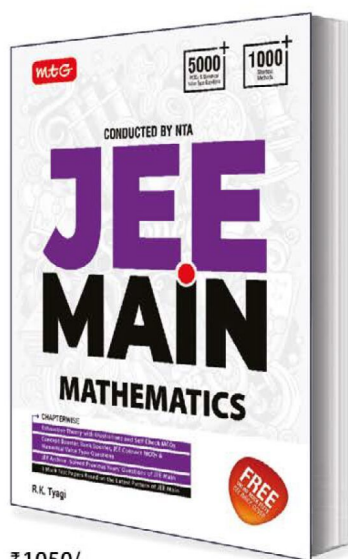
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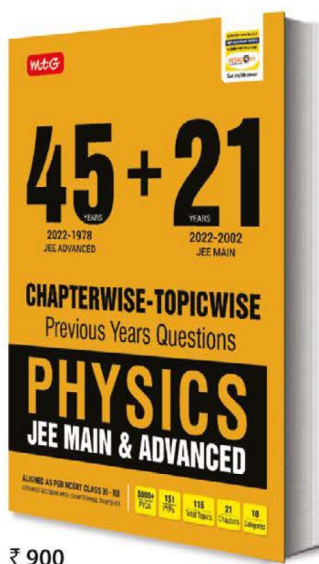
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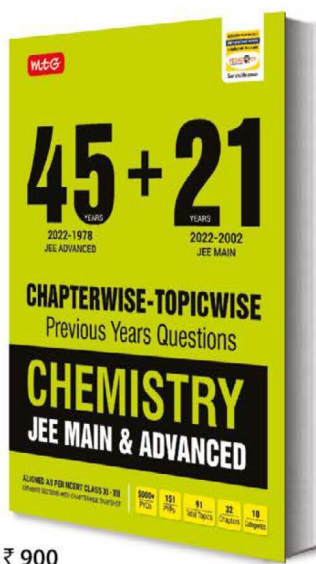


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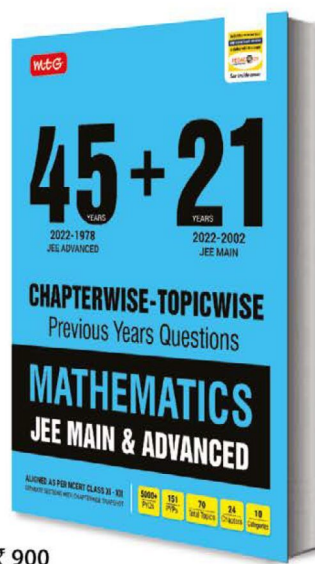
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PRACTICE PAPER 2023 JEE MAIN

Exam Dates

Session-2

06, 08, 10, 11, 12
April 2023

SECTION-A (MULTIPLE CHOICE QUESTIONS)

- Let S be the set of all (α, β) , $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha, \beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right)$ is equal to
(a) 3 (b) $3i$ (c) 1 (d) $2-i$
- Let z be a complex number such that $\left| \frac{z-i}{z+2i} \right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is
(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\frac{15}{4}$ (d) $2\sqrt{3}$
- If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{c}{a}, \frac{a}{b}$ and $\frac{b}{c}$ are in
(a) geometric progression
(b) arithmetic progression
(c) arithmetic-geometric progression
(d) None of these
- The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in \mathbb{R}, x \neq y)$ for which $A^T A = 3I_3$ is
(a) 4 (b) 3 (c) 2 (d) 6
- If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \dots + A^{20}$, then the sum of all the elements of the matrix M is
(a) 2020 (b) 480 (c) 2000 (d) 1020
- A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways of drawing 3 balls from the box, if atleast one black ball is included, is
(a) 36 (b) 42 (c) 56 (d) 64
- $\sum_{k=0}^{20} \binom{20}{k}^2$ is equal to
(a) ${}^{40}C_{21}$ (b) ${}^{41}C_{20}$ (c) ${}^{40}C_{20}$ (d) ${}^{40}C_{19}$
- The lowest integer which is greater than $\left(1 + \frac{1}{10^{100}} \right)^{10^{100}}$ is
(a) 2 (b) 1 (c) 3 (d) 4
- If the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ up to n^{th} term is 488 and the n^{th} term is negative, then
(a) $n = 60$ (b) n^{th} term is -4
(c) $n = 41$ (d) n^{th} term is $-4\frac{2}{5}$
- Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d . If for some positive integers $m, n, m \neq n$, $T_m = \frac{1}{n}$, and $T_n = \frac{1}{m}$, then $a-d$ equals
(a) $1/mn$ (b) 1 (c) 0 (d) $\frac{1}{m} + \frac{1}{n}$
- If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$ is equal to
(a) $2(b^2 + 4c)$ (b) $b^2 - 4c$
(c) $2(b^2 - 4c)$ (d) $b^2 + 4c$
- $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{15}$ (d) 0

13. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + c$,

where c is a constant of integration, then the ordered pair (a, b) is equal to

- (a) $(1, -3)$ (b) $(3, 1)$ (c) $(1, 3)$ (d) $(-1, 3)$

14. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to

(Here C is a constant of integration)

(a) $\log_e \frac{|x^3 + 1|}{x^2} + C$ (b) $\frac{1}{2} \log_e \frac{(x^3 + 1)^2}{|x^3|} + C$

(c) $\log_e \left| \frac{x^3 + 1}{x} \right| + C$ (d) $\frac{1}{2} \log_e \frac{|x^3 + 1|}{x^2} + C$

15. The differential equation of all circles passing through the origin and having their centres on the x -axis is

(a) $y^2 = x^2 + 2xy \frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy \frac{dy}{dx}$

(c) $x^2 = y^2 + xy \frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy \frac{dy}{dx}$

16. If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x + y - 2}{x - y}$ passes through the points $(2, 1)$ and $(k + 1, 2)$, $k > 0$, then

(a) $2 \tan^{-1} \left(\frac{1}{k} \right) = \log_e (k^2 + 1)$

(b) $\tan^{-1} \left(\frac{1}{k} \right) = \log_e (k^2 + 1)$

(c) $2 \tan^{-1} \left(\frac{1}{k+1} \right) = \log_e (k^2 + 2k + 2)$

(d) $2 \tan^{-1} \left(\frac{1}{k} \right) = \log_e \left(\frac{k^2 + 1}{k^2} \right)$

17. The x -coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as $(0, 1)$, $(1, 1)$ and $(1, 0)$ is

- (a) $2 - \sqrt{2}$ (b) $1 + \sqrt{2}$ (c) $1 - \sqrt{2}$ (d) $2 + \sqrt{2}$

18. In an isosceles triangle ABC , the vertex A is $(6, 1)$ and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of $\triangle ABC$, then $15(\alpha + \beta)$ is equal to

- (a) 39 (b) 41 (c) 51 (d) 63

19. Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is

- (a) $x^2 - 3xy + y^2 = 0$ (b) $x^2 + 4xy - y^2 = 0$
(c) $x^2 - 3xy - y^2 = 0$ (d) $x^2 + 3xy - y^2 = 0$

20. In a triangle PQR , the co-ordinates of the points P and Q are $(-2, 4)$ and $(4, -2)$ respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the $\triangle PQR$ is

- (a) $(-1, 0)$ (b) $(-2, -2)$
(c) $(0, 2)$ (d) $(1, 4)$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. If the lines $x - y + z = -1 = 3x - y + az + 1$ and $x + 4y - z - 1 = 0 = x + 2y + bz + 1$ are perpendicular, then $3ab + a - b + c = 0$, where c is _____.

22. Let P be an arbitrary point having sum of the squares of the distances from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to _____.

23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l , then the value of $3l^2$ is equal to _____.

24. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$, then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to _____.

25. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____.

26. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is _____.

27. Let $S = \{E_1, E_2, \dots, E_8\}$ be a sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every $n = 1, 2, \dots, 8$. Then the number of elements in the set $\left\{ A \subseteq S : P(A) \geq \frac{4}{5} \right\}$ is _____.

28. In $\triangle ABC$, the lengths of sides AC and AB are 12 cm and 5 cm respectively. If the area of $\triangle ABC$ is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of $\triangle ABC$, then the value of $2R + r$ (in cm) is equal to _____.

29. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in \mathbb{N}$ be the slopes of three line segments OA , OB and OC , respectively, where O is origin. If circumcentre of $\triangle ABC$ coincides with origin and its orthocentre lies on y -axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos \alpha \cos \beta \cos \gamma}\right)^2$ is equal to _____.

30. The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p , q , r and s , is equal to _____.

SOLUTIONS

1. (c): Given complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary, then

$$\frac{1-i\sin\alpha}{1+2i\sin\alpha} \times \frac{1-2i\sin\alpha}{1-2i\sin\alpha} = \frac{1-2\sin^2\alpha-3i\sin\alpha}{1+4\sin^2\alpha}$$

Since the obtained complex number is purely imaginary. So, real part will be zero.

$$\Rightarrow \frac{1-2\sin^2\alpha}{1+4\sin^2\alpha} = 0 \Rightarrow \sin^2\alpha = \frac{1}{2}, \quad \alpha = \frac{5\pi}{4}, \frac{7\pi}{4}$$

Now, $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real.

$$\Rightarrow \frac{1+i\cos\beta}{1-2i\cos\beta} \times \frac{(1+2i\cos\beta)}{(1+2i\cos\beta)} = \frac{1+3i\cos\beta-2\cos^2\beta}{1+4\cos^2\beta}$$

Since the obtained complex number is purely real. So, imaginary part will be zero.

$$\Rightarrow 3\cos\beta = 0 \Rightarrow \beta = \frac{3\pi}{2}$$

Now, $Z = \sin 2\alpha + i\cos 2\beta$, $Z = 1-i$ or $-1-i$

and $\frac{1}{iZ} = \frac{-iZ}{|Z|^2} = \frac{-iZ}{2}$; So, $iZ + \frac{1}{iZ} = iZ - \frac{iZ}{2} = \frac{iZ}{2}$

$$\therefore \sum \left(iZ + \frac{1}{iZ} \right) = \frac{i}{2} \sum Z = \frac{i}{2} (-2i) = 1$$

2. (b): Let $z = x + iy$

Now, $\left| \frac{z-i}{z+2i} \right| = 1$ [Given]

$$\Rightarrow \left| \frac{x+iy-i}{x+iy+2i} \right| = 1 \Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$$

$$\Rightarrow -2y + 1 = 4y + 4$$

$$\Rightarrow 6y = -3 \Rightarrow y = -\frac{1}{2}; \text{ Also, } |z| = \frac{5}{2} \quad [\text{Given}]$$

$$\Rightarrow x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6 \Rightarrow x = \pm\sqrt{6}$$

$$\therefore z = \pm\sqrt{6} - \frac{i}{2}; \text{ So, } |z+3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}} = \frac{7}{2}$$

$$3. (b): \text{ Given } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow 2a^2c = bc^2 + ab^2 \Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c} \Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c} \in \text{A.P.}$$

$$4. (a): \text{ Given, } A^T A = 3I_3$$

$$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow 8x^2 = 3, 6y^2 = 3$$

$$\Rightarrow x^2 = \frac{3}{8}, y^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{3}{8}}, y = \pm\sqrt{\frac{1}{2}}$$

$\therefore$ Required number of matrices is 4.

5. (a): Clearly,

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1+2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1+2+3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

and so on.

$$\text{Thus, we have } A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } M = \sum_{n=1}^{20} A^n = \begin{bmatrix} \sum_{n=1}^{20} 1 & \sum_{n=1}^{20} n & \sum_{n=1}^{20} \frac{n(n+1)}{2} \\ 0 & \sum_{n=1}^{20} 1 & \sum_{n=1}^{20} n \\ 0 & 0 & \sum_{n=1}^{20} 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

Now, required sum = $20 + 210 + 1540 + 20 + 210 + 20 = 2020$

6. (d): The number of ways of drawing 1 black and 2 non black balls is

$$\binom{3}{1} \cdot \binom{6}{2} = 3 \cdot 15 = 45$$

The number of ways of drawing 2 black and 1 non-black ball is $\binom{3}{2} \cdot \binom{6}{1} = 3 \cdot 6 = 18$

The number of ways of drawing 3 black balls is $\binom{3}{3} = 1$

$\therefore$ The desired number is $45 + 18 + 1 = 64$.

7. (c): We know that

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n \quad \dots(i)$$

$$\text{Also, } (x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \dots + {}^nC_n \dots(ii)$$

Multiply (i) & (ii), we get

$$(1+x)^{2n} = ({}^nC_0 + {}^nC_1 x + \dots + {}^nC_n x^n) ({}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_n)$$

Comparing coefficient of x^n both sides, we get

$${}^{2n}C_n = {}^nC_0^2 + {}^nC_1^2 + {}^nC_2^2 + \dots + {}^nC_n^2$$

$$\Rightarrow \sum_{r=0}^n ({}^nC_r)^2 = {}^{2n}C_n \therefore \sum_{k=0}^{20} ({}^{20}C_k)^2 = {}^{40}C_{20} \quad [\because n=20]$$

8. (c): We have, $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$

Let $10^{100} = n$

$$\Rightarrow \left(1 + \frac{1}{n}\right)^n = {}^nC_0 + {}^nC_1 \left(\frac{1}{n}\right) + {}^nC_2 \left(\frac{1}{n}\right)^2 + {}^nC_3 \left(\frac{1}{n}\right)^3 + \dots$$

$$= 1 + 1 + \frac{n(n-1)}{2n^2} + \frac{n(n-1)(n-2)}{6n^3} + \dots \Rightarrow \left(1 + \frac{1}{n}\right)^n > 2$$

$$\text{Also, } \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e < 3$$

9. (b): Here $a = 20$, $d = \frac{-2}{5}$

It is given that, $S_n = 488$

$$\Rightarrow \frac{n}{2} \left[2 \times 20 + (n-1) \left(\frac{-2}{5} \right) \right] = 488 \Rightarrow \frac{n}{5} [100 - n + 1] = 488$$

$$\Rightarrow n^2 - 101n + 2440 = 0 \Rightarrow n = 61, 40$$

Also, we have, $T_n < 0$

$$\Rightarrow 20 + (n-1) \left(\frac{-2}{5} \right) < 0$$

$$\Rightarrow 100 - 2n + 2 < 0 \Rightarrow 2n > 102 \Rightarrow n > 51$$

$$\text{So, } n = 61 \therefore T_n = 20 + 60 \times \left(\frac{-2}{5} \right) = 20 - 24 = -4$$

$$\mathbf{10. (c):} T_m = a + (m-1)d = \frac{1}{n} \quad \dots(i)$$

$$T_n = a + (n-1)d = \frac{1}{m} \quad \dots(ii)$$

$$\text{Now } T_m - T_n = \frac{1}{n} - \frac{1}{m} = (m-n)d$$

$$\Rightarrow d = \frac{1}{mn} \text{ and } a = \frac{1}{mn} \therefore a - d = 0$$

11. (c): We have, α, β are the roots of $x^2 + bx + c = 0$

$$\Rightarrow \alpha = \frac{-b + \sqrt{b^2 - 4c}}{2}, \beta = \frac{-b - \sqrt{b^2 - 4c}}{2}$$

$$\text{Also, } x^2 + bx + c = 0 \quad \dots(i)$$

$$\Rightarrow 2x + b = 0 \quad [\text{Differentiating both sides w.r.t. } x]$$

$$\text{Now, } \lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2} \left(\frac{0}{0} \text{ Form} \right)$$

$$= \lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} \cdot 2(2x+b) - 2(2x+b)}{2(x-\beta)}$$

[By L'Hospital Rule]

$$= \lim_{x \rightarrow \beta} \frac{2(2x+b) \cdot [e^{2(x^2+bx+c)} - 1]}{2(x-\beta)} \quad \left(\frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow \beta} \frac{(2x+b) \cdot e^{2(x^2+bx+c)} \cdot 2(2x+b) + (e^{2(x^2+bx+c)} - 1) \cdot 2}{1}$$

[By L'Hospital Rule]

$$= e^0 \cdot 2(2\beta + b)^2 = 2 \left[2 \left(\frac{-b - \sqrt{b^2 - 4c}}{2} \right) + b \right]^2 = 2(b^2 - 4c)$$

$$\mathbf{12. (a):} \lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3} \quad \left(\frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(\sin x) 2x}{3x^2} \quad [\text{Applying L'Hospital Rule}]$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \times \frac{2}{3} = \frac{2}{3}$$

$$\mathbf{13. (c):} \text{ Let } I = \int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$$

$$= \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}} dx$$

$$\text{Put } \sin x + \cos x = t \Rightarrow (\cos x - \sin x) dx = dt$$

$$\therefore I = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1} \frac{t}{3} + c = \sin^{-1} \left(\frac{\sin x + \cos x}{3} \right) + c$$

$$\text{So, } a = 1, b = 3$$

14. (c): Let $I = \int \frac{2x^3 - 1}{x^4 + x} dx = \int \frac{2x - 1/x^2}{x^2 + 1/x} dx$

[By dividing numerator and denominator by x^2]

Put $x^2 + \frac{1}{x} = t \Rightarrow \left(2x - \frac{1}{x^2}\right) dx = dt$

$\therefore I = \int \frac{dt}{t} = \log_e(t) + C = \log_e\left(x^2 + \frac{1}{x}\right) + C = \log_e\left|\frac{x^3 + 1}{x}\right| + C$

15. (a): General equation of all required circles is $(x - h)^2 + (y - 0)^2 = h^2$

$\Rightarrow (x - h)^2 + y^2 = h^2$ (i), where h is parameter.

Differentiating, we get $2(x - h) + 2y \frac{dy}{dx} = 0$

$\Rightarrow h = x + y \frac{dy}{dx}$

To eliminate h , put value of h in equation (i), we get

$y^2 = x^2 + 2xy \frac{dy}{dx}$

16. (a): Given $\frac{dy}{dx} = \frac{x + y - 2}{x - y}$

Put $x = X + h$ and $y = Y + k$

$\therefore \frac{dY}{dX} = \frac{X + Y + h + k - 2}{X + h - Y - k} = \frac{X + Y + (h + k - 2)}{X - Y + (h - k)}$

$\therefore \frac{dY}{dX} = \frac{X + Y}{X - Y}$, which is a homogenous differential equation.

Put $Y = VX \therefore V + \frac{dV}{dX} X = \frac{X + VX}{X - VX} = \frac{1 + V}{1 - V}$

$\Rightarrow \frac{X \cdot dV}{dX} = \frac{1 + V^2}{1 - V} \Rightarrow \frac{dV(1 - V)}{1 + V^2} = \frac{dX}{X}$

Integrating both sides, we get $\int \left(\frac{1 - V}{1 + V^2} \right) dV = \int \frac{dX}{X}$

$\Rightarrow \tan^{-1} V - \frac{1}{2} \log(1 + V^2) = \log X + c$

$\Rightarrow \tan^{-1} \left(\frac{Y}{X} \right) = \log \sqrt{X^2 + Y^2} + c$

$\Rightarrow \tan^{-1} \left(\frac{y - 1}{x - 1} \right) = \log \sqrt{(x - 1)^2 + (y - 1)^2} + c$

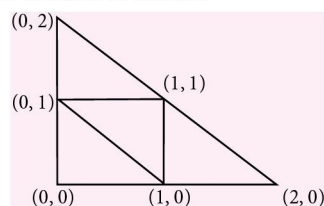
Since, it is passing through $(2, 1) \Rightarrow c = 0$

Also, it is passing through $(k + 1, 2)$

$\therefore \tan^{-1} \left(\frac{1}{k} \right) = \log \sqrt{k^2 + 1}$

$\Rightarrow 2 \tan^{-1} \left(\frac{1}{k} \right) = \log_e(k^2 + 1)$

17. (a): The triangle whose side's midpoints are given to be $(0, 1)$, $(1, 0)$ and $(1, 1)$ happen to be a right angled triangle with vertices as shown.



x -coordinate of incentre

$$= \frac{ax_1 + bx_2 + cx_3}{a + b + c} = \frac{2 \times 2 + 2\sqrt{2} \times 0 + 2 \times 0}{2 + 2 + 2\sqrt{2}}$$

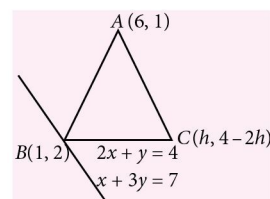
$$= \frac{4}{4 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$$

18. (c): Point B is Intersection point of line $x + 3y = 7$ and $2x + y = 4$
 Point B is $(1, 2)$

$\therefore \Delta$ is isosceles with base BC

$\therefore AB = AC$

$\Rightarrow \sqrt{25 + 1} = \sqrt{(6 - h)^2 + (2h - 3)^2}$





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$$\Rightarrow \sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$$

$$\Rightarrow h = \frac{19}{5} \text{ or } h = 1$$

$$\text{Point C is } \left(\frac{19}{5}, -\frac{18}{5} \right)$$

Centroid of triangle ABC is;

$$\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3} \right) = \left(\frac{54}{15}, \frac{-3}{15} \right) \therefore \alpha = \frac{54}{15}, \beta = \frac{-3}{15}$$

$$\therefore 15(\alpha + \beta) = 51.$$

19. (d)

20. (b): Let A is mid point of PQ.

So, coordinates of point A

$$= \left(\frac{-2+4}{2}, \frac{4-2}{2} \right) \text{ i.e., } (1, 1)$$

$$\text{Slope of PQ} = \frac{4+2}{-2-4} = -1$$

$\Rightarrow$ Slope of perpendicular bisector of PQ = 1

So, equation of perpendicular bisector of PQ is

$$(y - 1) = 1(x - 1)$$

$$\Rightarrow x - y = 0 \quad \dots(i)$$

Also, equation of perpendicular bisector of PR is

$$2x - y + 2 = 0 \quad \dots(ii)$$

Solving (i) and (ii), we get $x = -2$ and $y = -2$

$\Rightarrow$ Centre of circumcircle of ΔPQR is $(-2, -2)$.

21. (5): The d.r's of the first line are given by

$$l - m + n = 0, 3l - m + an = 0$$

$$\Rightarrow l : m : n = 1 - a : 3 - a : 2$$

The d.r's of the second line are given by $l + 4m - n = 0, l + 2m + bn = 0$

$$\therefore l : m : n = 4b + 2 : -(b + 1) : -2$$

The lines are perpendicular

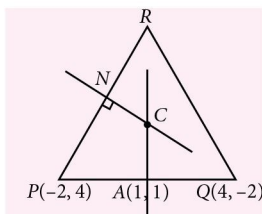
$$(1 - a)(4b + 2) - (3 - a)(b + 1) - 4 = 0$$

$$\Rightarrow 3ab + a - b + 5 = 0$$

22. (0): According to question,

$$\left(\frac{x+y+z}{\sqrt{3}} \right)^2 + \left(\frac{lx-nz}{\sqrt{l^2+n^2}} \right)^2 + \left(\frac{x-2y+z}{\sqrt{6}} \right)^2 = 9$$

$$\Rightarrow x^2 \left(\frac{1}{2} + \frac{l^2}{l^2+n^2} \right) + y^2 + z^2 \left(\frac{1}{2} + \frac{n^2}{l^2+n^2} \right) + 2zx \left(\frac{1}{2} - \frac{ln}{l^2+n^2} \right) = 9$$



Since, we have $x^2 + y^2 + z^2 = 9$

$$\Rightarrow \frac{l^2}{l^2+n^2} = \frac{1}{2} \Rightarrow l = \pm n. \text{ Also, } \frac{ln}{l^2+n^2} = \frac{1}{2}$$

$$\Rightarrow l - n = 0$$

23. (2): We have, $\vec{a} \times \vec{b} = \vec{c} \Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2$

$$\text{Also, } \vec{a} \times \vec{c} = -2\hat{i} + \hat{j} + \hat{k}$$

$$\text{Now, length of projection of } \vec{b} \text{ on } \vec{a} \times \vec{c} = \frac{|\vec{b} \cdot (\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|}$$

$$= \frac{|-2|}{|\vec{a} \times \vec{c}|} = \frac{2}{\sqrt{4+1+1}} = \frac{2}{\sqrt{6}}$$

$$\Rightarrow l = \frac{2}{\sqrt{6}} \Rightarrow 3l^2 = 3 \times \frac{4}{6} = 2$$

24. (28): Given, $\vec{c} = \lambda(\vec{a} \times \vec{b})$,

$$\text{where, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow \vec{c} = \lambda(3\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{Now, } \vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$$

$$\Rightarrow \lambda(3 - 2 + 3) = 8 \Rightarrow \lambda = 2 \Rightarrow \vec{c} = 2(3\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{Hence, } \vec{c} \cdot (\vec{a} \times \vec{b}) = 2 \begin{vmatrix} 3 & -2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 28$$

25. (2): Let x_{10} was by mistake written as 25 but $x_{10} = 15$

$$\Rightarrow \frac{x_1 + x_2 + \dots + x_9 + 25}{10} = 15 \Rightarrow x_1 + x_2 + \dots + x_9 = 125$$

$$\text{Correct mean} = \frac{125 + 15}{10} = 14$$

Given, incorrect variance = 15

$$\Rightarrow \frac{x_1^2 + x_2^2 + \dots + x_9^2 + 25^2}{10} - 15^2 = 15$$

$$\Rightarrow \sum_{i=1}^9 x_i^2 = 2400 - 625 = 1775$$

$$\text{So, correct variance} = \frac{\sum_{i=1}^9 x_i^2 + 15^2}{10} - 14^2$$

$$= \frac{1775 + 225}{10} - 196 = 4 \Rightarrow \text{S.D.} = 2$$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (a) 2. (c) 3. (c) 4. (d) 5. (b)
6. (a) 7. (a, c) 8. (a, b, c) 9. (a, c) 10. (a, b, c)
11. (a, b, c, d) 12. (a, b, c) 13. (c) 14. (b)
15. (d) 16. (b) 17. (13) 18. (6) 19. (1)
20. (81)

26. (96): Here, Mean = np and Variance = npq

Now, $np + npq = 82.5 \Rightarrow np(1 + q) = 82.5$

and $(np)(npq) = 1350 \Rightarrow n^2 p^2 q = 1350$

From (i) and (ii), $\frac{q}{(1+q)^2} = \frac{1350}{(82.5)^2} = \frac{24}{121}$

$$\Rightarrow 24(1+q)^2 = 121q$$

$$\Rightarrow 24q^2 - 73q + 24 = 0 \Rightarrow q = \frac{3}{8} \text{ or } q = \frac{8}{3}$$

$$\Rightarrow q = \frac{3}{8} \text{ and } p = 1 - \frac{3}{8} = \frac{5}{8} \therefore n = 96$$

27. (19): Sample space $S = \{E_1, E_2, \dots, E_8\}$

$$P(E_1) = \frac{1}{36}, P(E_2) = \frac{2}{36}, \dots, P(E_8) = \frac{8}{36}$$

$$\text{Let } P(A) = \frac{\text{sum of element of any subset } (\lambda)}{36} = \frac{\lambda}{36}$$

$$\text{Since } P(A) \geq \frac{4}{5}; \frac{\lambda}{36} \geq \frac{4}{5} \Rightarrow \lambda \geq \frac{144}{5}$$

$\lambda = 29, 30, 31, \dots, 36$.

If one of the number from $(1, 2, 3, \dots, 8)$ is removed then total $\lambda \geq 29$ by 7 ways.

If we remove terms 2 or 3, we get 11 more combinations.

When consider all numbers, then such ways is 1.

$\therefore$ Total number of different set A is $7 + 11 + 1 = 19$.

28. (15): Let $\Delta = \text{Area of } \triangle ABC = 30 \text{ cm}^2$

$$\Rightarrow \frac{1}{2} bc \sin A = 30$$

$$\Rightarrow \frac{1}{2} \times 12 \times 5 \sin A = 30$$

$$\Rightarrow \sin A = 1 \Rightarrow A = \frac{\pi}{2}$$

$$\therefore a = \sqrt{12^2 + 5^2} = 13 \text{ cm}$$

$$\text{Now, } R = \frac{a}{2 \sin A} = \frac{13}{2 \times 1} = \frac{13}{2} \text{ cm}$$

$$\text{Now, } S = \frac{a+b+c}{2} = 15 \text{ cm and } r = \frac{\Delta}{S} = \frac{30}{15} = 2 \text{ cm}$$

$$\text{Hence, } 2R + r = 2 \times \frac{13}{2} + 2 = 15 \text{ cm}$$

29. (144): Let $A(r \cos \alpha, r \sin \alpha)$, $B(r \cos \beta, r \sin \beta)$ and $C(r \cos \gamma, r \sin \gamma)$.

Since, circumcentre and orthocentre both lies on y -axis.

$\therefore$ Centroid also lies on y -axis.

$$\Rightarrow \cos \alpha + \cos \beta + \cos \gamma = 0$$

$$\therefore \cos^3 \alpha + \cos^3 \beta + \cos^3 \gamma = 3 \cos \alpha \cdot \cos \beta \cdot \cos \gamma \dots(i)$$

Now, $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 4(\cos^3 \alpha + \cos^3 \beta + \cos^3 \gamma)$

$$\Rightarrow \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos^3 \alpha + \cos^3 \beta + \cos^3 \gamma} = 4 \dots(ii)$$

Using (i) and (ii), we get

...(i)

...(ii)

$$\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos \alpha \cdot \cos \beta \cdot \cos \gamma} \right)^2 = 12^2 = 144$$

30. (9): There are total 9 compound propositions. out of which

$$p \vee r \vee \sim s, \sim p \vee \sim r \vee \sim s, \sim p \vee q \vee \sim s,$$

$$q \vee r \vee \sim s, q \vee \sim r \vee \sim s, \sim p \vee \sim q \vee \sim s$$

contain $\sim S$. So if we assume S as False then $\sim S$ becomes True and all these 6 propositions become True.

Now there are 3 proposition left which is,

$$p \vee r \vee s, p \vee \sim q \vee s, \sim p \vee \sim r \vee s$$

So, if we assign p as True and r as False then all the three propositions become True.

Hence, maximum number of compound proposition out of 9 propositions that can be made simultaneously True is 9.



SAMURAI SUDOKU



Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9×9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3×3 box should contain one of each digit.

The puzzle has a unique answer.

5	9		3		2	1				6	3		7		4	8		
2			5		8				3		7		2		4		6	
7								8			4						7	
	4						2				8						1	
5				1					7		1			6			3	
	7						8				2						9	
9							2			8							9	
4			3		2			6			2			7		9	8	
	2	5		8		7	4			2			9	5		3	7	2
						7		5										
						9				5								
						9		4										
	5	2		3		9	1			6			5	8		1	2	9
3			7		8			5			3			4		8		6
7							8				7							4
	3						9					6					4	
9				7			2				9			5				2
	1						5					8					3	
6											8							5
4			1	2			9				5		1	7				8
	2	5		4		6	7					7	1		9	4	2	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Resonance has announced admission for all courses for the session 2023-24

Kota's Resonance Institute, known for preparing for competitive examinations from 22 years, has announced the commencement of admission for class V to XII and XII pass students for the upcoming academic session 2023-24. Yearlong class programs of the institute can be taken by class 11th, 12th and class 12th pass students who want to prepare for IIT, NEET (UG), JEE (Mains), Olympiad or Board exams in the coming year while class 5th to 10th students can also prepare for their Olympiad, Board and School examinations by joining the Pre Foundation Division of the Institute.

One, Two, Three and Four Year Programs

The process of admission has started in all the programs run by the Institute *i.e.*, one, two, three and four year programs. Students from classes 5 to 8 can take admission in the one year program. Class 9 students can opt for 1-year or 4-year program while class 10 students also have the option of going in for 1-year or 3-year program. The institute has also announced a 2-year program for students taking admission in class 11th from this year, in which students can make great savings in terms of fees of up to Rs 2,50,000 if they take early admission.

Maximum Benefits on Early Admissions

The Institute has kept the lowest fee for the most early admissions, in fact, the fee for confirmed admission (in February) is much less than the last year's fee structure. Hence, students should take maximum advantage of this scheme by ensuring admission in the month of February itself.

Resonance National Entrance Test (ResoNET)

Students from both the yearlong and pre-foundation divisions can get the scholarship of up to 100% by appearing for Resonance's National Entrance Test (ResoNET). This exam is conducted in both offline and online mode. In addition to all the 44 study centers of Resonance, this offline exam is also held at 67 test centers. ResoNET offline exam

for yearlong courses is of 3 hours and online exam is of 2 hours, while in Pre Foundation Division, the time for both online and offline exams of ResoNET is 1.5 hours.

Application and Fee Submission Process

For admission in the courses of the institute, students can fill the form through online or offline medium. The online form can be filled by visiting the institute's website www.resonance.ac.in.

Both online and offline options are available to pay the fees. Fees can be deposited on the website of the institute www.resonance.ac.in or by visiting the study centers. Cards, UPI, Net banking etc., all the options are available for online fee deposit.

There is a provision of **special concession** on lump sum fee deposit, however, students/ parents can also deposit the fees in EMIs or equated monthly installments.

Resonance has dominated the competitive examinations

The students of the institute had recently performed brilliantly in JEE Advanced 2022 and NEET (UG) 2022. Institute's student Karthikeya Polisetty secured All India Rank 6 while Dheeraj Kurukunda secured AIR 8 in JEE Advanced 2022 result. 8 students of the institute made it to the top 60 ranks from the overall ranking. On the other hand, in the NEET (UG) 2022 result, Vrajesh V Shetty, a student of the institute, has achieved All India Rank 13 and Rishit Agarwal has achieved All India Rank 27, performing remarkably. The institute has a selection ratio of 86.92% in NEET (UG) 2022.

Resonance has placed over 50,000 students in IIT/JEE (Advanced) in 21 years, over 2.4 lakh in JEE (Main)/AIEEE in 13 years and over 19,500 in NEET(UG)/AIPMT in 10 years.

More information about the institute's courses and admission process can be obtained on 0744-2777777.



CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

Exam on
21st to 31st
May 2023

SERIES - VI Vector Algebra & Three Dimensional Geometry

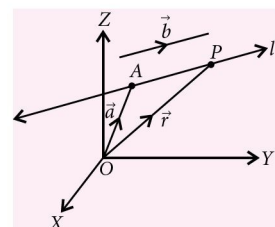
Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time Allowed : 45 Minutes

Maximum Marks : 200

Multiple Choice Questions (MCQs)

- If direction cosines of two lines are proportional to $(2, 3, -6)$ and $(3, -4, 5)$, then the acute angle between them is
(a) $\cos^{-1}\left(\frac{49}{36}\right)$ (b) $\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$
(c) 96° (d) $\cos^{-1}\left(\frac{18}{35}\right)$
- If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a} + \vec{b}| < 1$, then
(a) $\theta = \frac{\pi}{2}$ (b) $\theta < \frac{\pi}{3}$
(c) $\pi \geq \theta > \frac{2\pi}{3}$ (d) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$
- If $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is
(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0
- If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed
(a) 4 (b) 9 (c) 8 (d) 6
- Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ is equal to
(a) 2 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 14
- $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to
(a) 12 (b) 3 (c) 8 (d) 4
- If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$, then $\lambda + \mu$ is equal to
(a) 0 (b) 1 (c) 2 (d) 3
- If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $\hat{c}\hat{i} + \hat{c}\hat{j} + b\hat{k}$ are coplanar, then $c^2 =$
(a) a^2b (b) ab (c) a^2b^2 (d) $\sqrt{ab}$
- In the figure, $\vec{a}$ be the position vector of the point A with respect to the origin O. l is a line parallel to a vector $\vec{b}$. The vector equation of line l is
(a) $\vec{r} = \lambda(\vec{a} \times \vec{b})$ (b) $\vec{r} = \lambda\vec{a} - \vec{b}$
(c) $\vec{r} = \vec{a} + \lambda\vec{b}$ (d) $\vec{r} = \lambda(\vec{a} \cdot \vec{b})$
- If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is
(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$
- The value of p , so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ intersect at right angle, is
(a) $\frac{10}{11}$ (b) $\frac{70}{11}$ (c) $\frac{10}{7}$ (d) $\frac{70}{9}$
- The direction cosines of a line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are





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- (a) $(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$
 (b) $(x_2 + x_1), (y_2 + y_1), (z_2 + z_1)$
 (c) $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$
 (d) $\frac{x_2 + x_1}{PQ}, \frac{y_2 + y_1}{PQ}, \frac{z_2 + z_1}{PQ}$
13. The equation of a line passing through the point $(-3, 2, -4)$ and equally inclined to the axes are
 (a) $x - 3 = y + 2 = z - 4$ (b) $x + 3 = y - 2 = z + 4$
 (c) $\frac{x+3}{1} = \frac{y-2}{2} = \frac{z+4}{3}$ (d) None of these
14. The vector and cartesian equations of the line that passes through the points $(3, -2, -5)$ and $(3, -2, 6)$ are
 (a) $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k}; x - 3 = y + 2 = \frac{z+5}{11}$
 (b) $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k}; \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$
 (c) $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k}); x - 3 = y - 2 = \frac{z+5}{11}$
 (d) $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k}); \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$
15. The angle between the lines $\frac{x-5}{1} = \frac{y-7}{2} = \frac{z-11}{3}$ and $\frac{x}{2} = \frac{y+3}{2} = \frac{z-4}{-2}$ is
 (a) 90° (b) 0°
 (c) 180° (d) None of these
16. Equation of a line passing through $(1, 2, -3)$ and parallel to the line $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$ is
 (a) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$ (b) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{-3}$
 (c) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-4}{-3}$ (d) None of these
17. The equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$ is
 (a) $5x + 4y - z = 7$ (b) $5x - 4y + z = 7$
 (c) $-5x + 4y - z = 7$ (d) $5x - 4y - z = 7$
18. The shortest distance between the lines $\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ is
 (a) 84 (b) 14 (c) 21 (d) 16
19. The distance between the lines $\frac{x-4}{5} = \frac{y+1}{2} = \frac{z}{1}$ and $\frac{x-1}{5} = \frac{y-2}{2} = \frac{z-3}{1}$ is
 (a) $5\sqrt{129}$ (b) $\frac{\sqrt{129}}{5}$ (c) $\sqrt{\frac{129}{10}}$ (d) $\sqrt{\frac{129}{5}}$
20. Four points $(0, -1, -1)$ $(-4, 4, 4)$ $(4, 5, 1)$ and $(3, 9, 4)$ are coplanar. Find the equation of the plane containing them.
 (a) $5x + 7y + 11z - 4 = 0$ (b) $5x - 7y + 11z + 4 = 0$
 (c) $5x - 7y - 11z - 4 = 0$ (d) $5x + 7y - 11z + 4 = 0$
21. The lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{k} = \frac{z+1}{7}$ are coplanar if $k =$
 (a) 4 (b) -4 (c) 2 (d) -2
22. The angle between the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 4$ and $\vec{r} \cdot (-\hat{i} + \hat{j} + 2\hat{k}) = 9$ is
 (a) 30° (b) 60°
 (c) 45° (d) None of these
23. The distance of the origin from the plane through the points $(1, 1, 0)$, $(1, 2, 1)$ and $(-2, 2, -1)$ is
 (a) $\frac{3}{\sqrt{11}}$ (b) $\frac{5}{\sqrt{22}}$ (c) 3 (d) $\frac{4}{\sqrt{22}}$
24. If the sum of two unit vectors is also a unit vector, then the angle between the two vectors is
 (a) $\pi/3$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$
25. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $\vec{A} \equiv (1, a, a^2)$, $\vec{B} \equiv (1, b, b^2)$, $\vec{C} \equiv (1, c, c^2)$ are non-coplanar, then the value of abc is equal to
 (a) -2 (b) 0 (c) 1 (d) -1
26. Unit vectors $\vec{\alpha}$ and $\vec{\beta}$ are inclined at an angle ϕ and $|\vec{\alpha} - \vec{\beta}| < 1$, if $0 \leq \phi \leq \pi$, then ϕ may belong to
 (a) $\left[0, \frac{\pi}{3}\right]$ (b) $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$
 (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) None of these

27. If the line through the points $A(4, 1, 2)$, $B(5, \lambda, 0)$ is parallel to the line joining the points $C(2, 1, 1)$ and $D(3, 3, -1)$ then the value of λ equals

(a) 1 (b) 3 (c) 2 (d) -3

28. The coordinates of the foot of the perpendicular from $(1, 8, 4)$ to the line joining the points $(0, -1, 3)$ and $(2, -3, -1)$ are

(a) $\left(\frac{5}{3}, \frac{-2}{3}, \frac{19}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{2}{3}, \frac{19}{3}\right)$
(c) $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ (d) None of these

29. Equation of line passing through $(3, 4, 5)$ and perpendicular to the plane $2x + 3y + z + 5 = 0$ is

(a) $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-1}{5}$ (b) $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{1}$
(c) $\frac{x+2}{3} = \frac{y+3}{4} = \frac{z+1}{5}$ (d) None of these

30. The length of the perpendicular from $O(0,0,0)$ to the plane passing through a point $\vec{c}$ and containing the line $\vec{r} = \vec{b} + \mu\vec{a}$ is

(a) $\frac{[\vec{a} \vec{b} \vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$ (b) $\frac{[\vec{c} \vec{b} \vec{a}]}{|\vec{b} \times \vec{a} + \vec{a} \times \vec{c}|}$
(c) $\frac{[\vec{a} \vec{b} \vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c}|}$ (d) $\frac{[\vec{a} \vec{b} \vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$

31. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is

(a) $\vec{r} = (3\hat{i} + 7\hat{j} + 2\hat{k}) + \mu(5\hat{i} + 4\hat{j} - 6\hat{k})$
(b) $\vec{r} = (5\hat{i} + 4\hat{j} - 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$
(c) $\vec{r} = (5\hat{i} - 4\hat{j} - 6\hat{k}) + \mu(3\hat{i} - 7\hat{j} - 2\hat{k})$
(d) $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$

32. Equation of the plane whose intercepts are 1, 2, 3 is

(a) $6x + 2y + 3z = 1$ (b) $x + y + z = 6$
(c) $6x + 3y + 2z = 6$ (d) $6x - 3y - 2z = 1$

33. The angle between the line

$\vec{r} = (2\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} + 5\hat{k}) = 4$ is

(a) $\sin^{-1}\left(\frac{11}{\sqrt{323}}\right)$ (b) $\sin^{-1}\left(\frac{22}{\sqrt{323}}\right)$
(c) $\sin^{-1}\left(\frac{22}{\sqrt{646}}\right)$ (d) None of these

34. Find the volume of the tetrahedron whose vertices are $A(3, 7, 4)$, $B(5, -2, 3)$, $C(-4, 5, 6)$ and $D(1, 2, 3)$.

(a) 12 cu. units (b) $\frac{23}{3}$ cu. units
(c) 15 cu. units (d) $\frac{46}{3}$ cu. units

Assertion & Reason Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- (a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false and Statement-II is true.

35. **Statement-I:** The length of projection of the vector $3\hat{i} - \hat{j} - 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} - 3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

Statement-II: The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a} \cdot \vec{b})}{|\vec{b}|}$.

36. **Statement-I:** If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 400$ and $|\vec{a}| = 4$, then $|\vec{b}| = 9$.

Statement-II: If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a} \times \vec{b})^2$ is equal to $(\vec{a})^2 (\vec{b})^2 - (\vec{a} \cdot \vec{b})^2$.

37. **Statement-I:** If the cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.

Statement-II: The cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

38. Consider the lines

$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}, L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}.$$

Statement-I : The distance of point (1, 1, 1) from the plane passing through the point (-1, -2, -1) and whose normal is perpendicular to both the lines L_1 and L_2 is $\frac{13}{5\sqrt{3}}$.

Statement-II : The unit vector perpendicular to both the lines L_1 and L_2 is $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$.

Matrix-Match

39. Match Column I with Column II and select the correct answer by choosing an appropriate option.

Column - I		Column - II	
(P)	If $ \vec{a} + \vec{b} = \vec{a} + 2\vec{b} $, then angle between $\vec{a}$ and $\vec{b}$ is	(1)	90°
(Q)	If $ \vec{a} + \vec{b} = \vec{a} - 2\vec{b} $, then angle between $\vec{a}$ and $\vec{b}$ is	(2)	obtuse
(R)	If $ \vec{a} + \vec{b} = \vec{a} - \vec{b} $, then angle between $\vec{a}$ and $\vec{b}$ is	(3)	0°
(S)	Angle between $\vec{a} \times \vec{b}$ and a vector perpendicular to the vector $\vec{c} \times (\vec{a} \times \vec{b})$ is	(4)	acute

P	Q	R	S
(a) 1	2	3	4
(b) 2	4	1	3
(c) 3	4	2	1
(d) 2	3	1	4

40. A plane π passes through the point (1, 1, 1) and is parallel to the vectors $b = (1, 0, -1)$ and $a = (-1, 1, 0)$. π meets the axis in A, B and C forming the tetrahedron OABC. Now match the following columns.

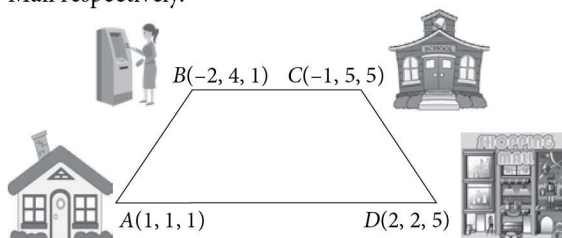
Column I		Column II	
(P)	Volume of the tetrahedron OABC in cubic units is	(1)	$\frac{9}{2}$
(Q)	Area of a face of the tetrahedron in square units is	(2)	$\frac{9\sqrt{3}}{2}$
(R)	Distance of the points, $(3\sqrt{3}/2, 3\sqrt{3}, 3)$ from the plane π in units is	(3)	$\sqrt{3}$
(S)	Distance of a face of the tetrahedron from the vertex is	(4)	3

	P	Q	R	S
(a) 3	1	4	2	
(b) 4	2	3	1	
(c) 1	1, 2	1	3, 4	
(d) 2	3	1	3	

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Ritika starts walking from her house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram, A, B, C and D represent the coordinates of House, ATM, School and Mall respectively.



41. Distance between House (A) and ATM (B) is
 (a) 3 units (b) $3\sqrt{2}$ units
 (c) $\sqrt{2}$ units (d) $4\sqrt{2}$ units
42. Distance between ATM (B) and School (C) is
 (a) $\sqrt{2}$ units (b) $2\sqrt{2}$ units
 (c) $3\sqrt{2}$ units (d) $4\sqrt{2}$ units
43. Distance between School (C) and Shopping mall (D) is
 (a) $3\sqrt{2}$ units (b) $5\sqrt{2}$ units
 (c) $7\sqrt{2}$ units (d) $10\sqrt{2}$ units
44. What is the total distance travelled by Ritika?
 (a) $4\sqrt{2}$ units (b) $6\sqrt{2}$ units
 (c) $8\sqrt{2}$ units (d) $9\sqrt{2}$ units
45. What is the extra distance travelled by Ritika in reaching the shopping mall?
 (a) $3\sqrt{2}$ units (b) $5\sqrt{21}$ units
 (c) $6\sqrt{2}$ units (d) $7\sqrt{2}$ units

Case II : Read the following passage and answer the questions from 46 to 50.

The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity

to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are (2, 3, 5) and (1, 4, 2) respectively.



46. If the line joining the positions of the helicopter and boat is perpendicular to the plane in which boat moves, then equation of plane is

- (a) $x - y + 3z = 2$ (b) $x + y + 3z = 2$
(c) $x - y + 3z = 3$ (d) $x + y + 3z = 3$

47. If the soldier decides to shoot the boat at given instant of time, where the distance measured in metres, then what is the distance that bullet has to travel?

- (a) $\sqrt{5}$ m (b) $\sqrt{8}$ m (c) $\sqrt{10}$ m (d) $\sqrt{11}$ m

48. If the speed of bullet is 30 m/sec, then how much time will the bullet take to hit the boat after the shot is fired?

- (a) 30 seconds (b) 1 second
(c) $\frac{1}{2}$ second (d) $\frac{\sqrt{11}}{30}$ seconds

49. At the given instant of time, the equation of line passing through the positions of helicopter and boat is

- (a) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{3}$ (b) $\frac{x-1}{1} = \frac{y-4}{-1} = \frac{z-2}{3}$
(c) $\frac{x}{1} = \frac{y}{1} = \frac{z}{-3}$ (d) $\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-2}{-3}$

50. At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat for the bullet to hit the boat if soldier shoots the bullet along the line whose equation is $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$?

- (a) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{3}{2}, \frac{5}{4}\right)$
(c) $\left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right)$ (d) None of these

SOLUTIONS

1. (b): We know that,

$$\begin{aligned}\cos \theta &= \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \\ &= \frac{|(2)(3) + (3)(-4) + (-6)(5)|}{\sqrt{2^2 + 3^2 + (-6)^2} \sqrt{3^2 + (-4)^2 + (5)^2}} \\ &= \frac{36}{7 \times 5\sqrt{2}} = \frac{18\sqrt{2}}{35} \Rightarrow \theta = \cos^{-1} \left(\frac{18\sqrt{2}}{35} \right)\end{aligned}$$

2. (c): $|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1 \Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} < 1$$

$$\Rightarrow \vec{a} \cdot \vec{b} < -\frac{1}{2} \Rightarrow |\vec{a}| |\vec{b}| \cos \theta < -\frac{1}{2}$$

$$\Rightarrow 1 \times 1 \times \cos \theta < -\frac{1}{2} \Rightarrow \cos \theta < -\frac{1}{2}$$

$$\Rightarrow -1 \leq \cos \theta < -\frac{1}{2} \Rightarrow \pi \geq \theta > \frac{2\pi}{3}$$

3. (a): Given, $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$

$$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 1 = 1 + 1 - 2|\vec{a}||\vec{b}|\cos \theta, \text{ where } \theta \text{ is angle between } \vec{a} \text{ and } \vec{b}.$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

4. (b): $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$

$$\begin{aligned}&= 2[|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2] - 2\Sigma(\vec{a} \cdot \vec{b}) = 2(1 + 1 + 1) - 2\Sigma(\vec{a} \cdot \vec{b}) \\ &= 6 - 2\Sigma(\vec{a} \cdot \vec{b}) \quad \dots(i)\end{aligned}$$

$$\text{But } (\vec{a} + \vec{b} + \vec{c})^2 \geq 0 \Rightarrow (1 + 1 + 1) + 2\Sigma(\vec{a} \cdot \vec{b}) \geq 0$$

$$\therefore 3 \geq -2\Sigma(\vec{a} \cdot \vec{b}) \quad \dots(ii)$$

From (i) and (ii), we get

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 6 + 3 = 9$$

5. (c): Given, $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$

According to the given condition

$$\frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} = \frac{\vec{w} \cdot \vec{u}}{|\vec{u}|} \Rightarrow \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$$

and $\vec{v}, \vec{w}$ are perpendicular to each other.

$$\therefore \vec{v} \cdot \vec{w} = 0$$

$$\text{Now, } |\vec{u} - \vec{v} + \vec{w}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u} \cdot \vec{v} + 2\vec{u} \cdot \vec{w} - 2\vec{v} \cdot \vec{w}$$

$$= 1 + 4 + 9 - 2\vec{u} \cdot \vec{v} + 2\vec{v} \cdot \vec{u}$$

$$\Rightarrow |\vec{u} - \vec{v} + \vec{w}|^2 = 14 \Rightarrow |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$$

6. (b): Given, $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$

$$\therefore |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$\Rightarrow 144 = 16 \cdot |\vec{b}|^2 \Rightarrow |\vec{b}|^2 = 9 \Rightarrow |\vec{b}| = 3$$

$$\mathbf{7. (a):} \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$$

$$\text{Now, } (\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\Rightarrow \lambda \vec{a} + \mu \vec{b} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-1)$$

$$\Rightarrow (\lambda + \mu)\hat{i} + (\lambda + \mu)\hat{j} + (\lambda)\hat{k} = -\hat{k}$$

On comparing, we get $\lambda = -1$ and $\lambda + \mu = 0$

8. (b): Since, the given vectors are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 - C_2$, we get

$$\begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0 \Rightarrow -1(ab - c^2) = 0 \Rightarrow c^2 = ab$$

9. (c)

10. (b): Since, the lines intersect they must have a point in common.

$$\text{Let } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda \text{ and } \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = 4\lambda + 1$$

$$\text{and } x = \mu + 3, y = 2\mu + k, z = \mu$$

$$\text{Now, consider } 2\lambda + 1 = \mu + 3, 3\lambda - 1 = 2\mu + k, 4\lambda + 1 = \mu$$

$$\text{On solving we get, } \lambda = -\frac{3}{2} \text{ and } \mu = -5$$

$$\therefore k = 3\lambda - 2\mu - 1 = 3\left(-\frac{3}{2}\right) - 2(-5) - 1 \Rightarrow k = \frac{9}{2}$$

11. (b): Equation of the given lines can be written in the standard form as

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \text{ and } \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

$\therefore$ Lines are perpendicular to each other

$$\therefore (-3)\left(\frac{-3p}{7}\right) + \left(\frac{2p}{7}\right)(1) + (2)(-5) = 0 \Rightarrow p = \frac{70}{11}$$

12. (c)

13. (b): Since, the line is equally inclined to the axes.

$$\therefore l = m = n$$

...(i)

The required equation of line is

$$\frac{x+3}{l} = \frac{y-2}{l} = \frac{z+4}{l} \quad [\text{using (i)}]$$

$$\Rightarrow \frac{x+3}{1} = \frac{y-2}{1} = \frac{z+4}{1} \Rightarrow x+3 = y-2 = z+4$$

14. (d): Let $\vec{a}$ and $\vec{b}$ be the position vectors of points $(3, -2, -5)$ and $(3, -2, 6)$, respectively.

$$\text{Then, } \vec{a} = 3\hat{i} - 2\hat{j} - 5\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$$

$\therefore$ Required equation is

$$\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda[(3\hat{i} - 2\hat{j} + 6\hat{k}) - (3\hat{i} - 2\hat{j} - 5\hat{k})]$$

$\Rightarrow \vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k})$, which is the required vector form.

So, equation of line passes through $(3, -2, -5)$

$$\text{and } (3, -2, 6) \text{ is } \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11},$$

which is the cartesian form of required line.

15. (a): The direction ratios of the lines are $a_1 = 1$, $b_1 = 2$, $c_1 = 3$ and $a_2 = 2$, $b_2 = 2$, $c_2 = -2$

$$\therefore \cos \theta = \frac{1 \cdot 2 + 2 \cdot 2 + 3 \cdot (-2)}{\sqrt{1+4+9}\sqrt{4+4+4}} = 0 \Rightarrow \theta = 90^\circ$$

16. (a): Since, the line is parallel to the line

$$\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$$

$\therefore$ DR's of the required line are $\langle 1, 3, 4 \rangle$

Hence, equation of the line passing through $(1, 2, -3)$

$$\text{with DR's } \langle 1, 3, 4 \rangle \text{ is } \frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$$

17. (d): Let the equation of the plane be $ax + by + cz = 1$

$$\text{Then, } 2a + 3b - 2c = 0 \quad \dots(i)$$

$$\text{and } a + 2b - 3c = 0 \quad \dots(ii)$$

[$\therefore$ Required plane is $\perp$ to both given planes]

Also, $(1, -1, 2)$ lies on the plane.

$$\therefore a - b + 2c = 1 \quad \dots(\text{iii})$$

Subtracting (iii) from (ii), we get $3b - 5c = -1 \quad \dots(\text{iv})$

Also, subtracting $2 \times (\text{ii})$ from (i), we get $-b + 4c = 0$

Putting $b = 4c$ in (iv), we get

$$3(4c) - 5c = -1 \Rightarrow 7c = -1 \Rightarrow c = -1/7$$

$$\text{Now, } b = -4/7$$

$$\text{From (i), } 2a - 12/7 + 2/7 = 0 \Rightarrow 2a = 10/7 \Rightarrow a = 5/7$$

So, equation of plane is

$$(5/7)x + (-4/7)y + (-1/7)z = 1 \Rightarrow 5x - 4y - z = 7$$

18. (b): Given, equation of lines are

$$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$$

$$\text{i.e., } \vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

$$\text{and } \vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$\text{Here, } \vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}, \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$$

$$\vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = (15\hat{i} + 29\hat{j} + 5\hat{k}) - (8\hat{i} - 9\hat{j} + 10\hat{k})$$

$$= 7\hat{i} + 38\hat{j} - 5\hat{k}$$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48)$$

$$= (24\hat{i} + 36\hat{j} + 72\hat{k})$$

$$\text{Now, shortest distance, } d = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{(7\hat{i} + 38\hat{j} - 5\hat{k}) \cdot (24\hat{i} + 36\hat{j} + 72\hat{k})}{\sqrt{(24)^2 + (36)^2 + (72)^2}}$$

$$= \frac{168 + 1368 - 360}{\sqrt{576 + 1296 + 5184}} = \frac{1176}{\sqrt{7056}} = \frac{1176}{84} = 14$$

19. (d)

20. (b): We shall find the equation of a plane passing through any three out of the given four points and show that the fourth point satisfies the equation.

Now, any plane passing through $(0, -1, -1)$ is

$$a(x - 0) + b(y + 1) + c(z + 1) = 0 \quad \dots(\text{i})$$

If it passes through $(-4, 4, 4)$ we have

$$a(-4) + b(5) + c(5) = 0 \Rightarrow -4a + 5b + 5c = 0 \quad \dots(\text{ii})$$

Also, if plane passes through $(4, 5, 1)$, we have

$$a(4) + b(6) + c(2) = 0 \Rightarrow 2a + 3b + c = 0 \quad \dots(\text{iii})$$

Solving (ii) and (iii) by cross multiplication method, we

$$\text{obtain } \frac{a}{-5} = \frac{b}{7} = \frac{c}{-11} = k$$

Putting these values in (i), we get

$$-5kx + 7k(y + 1) - 11k(z + 1) = 0 \Rightarrow -5x + 7y - 11z - 4 = 0$$

which is the required equation of plane.

Clearly, the fourth point namely $(3, 9, 4)$ satisfies this equation. Hence the given points are coplanar and the equation of plane containing points is

$$5x - 7y + 11z + 4 = 0$$

$$\text{21. (b): Given lines are } \frac{x-1}{2} = \frac{y-(-1)}{-3} = \frac{z-(-10)}{8}$$

$$\text{and } \frac{x-4}{1} = \frac{y-(-3)}{k} = \frac{z-(-1)}{7}$$

$\therefore$ These lines are coplanar.

$$\therefore \begin{vmatrix} 4-1 & -3-(-1) & -1-(-10) \\ 2 & -3 & 8 \\ 1 & k & 7 \end{vmatrix} = 0$$

$$\Rightarrow 3(-21 - 8k) + 2(14 - 8) + 9(2k + 3) = 0$$

$$\Rightarrow 6k = -24 \therefore k = -4.$$

22. (b): Angle between two planes will be the angle between their normals.

$$\therefore \cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|} = \frac{|(\hat{i} + 2\hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + 2\hat{k})|}{|\hat{i} + 2\hat{j} + \hat{k}||-\hat{i} + \hat{j} + 2\hat{k}|}$$

$$\Rightarrow \cos \theta = \frac{|-1 + 2 + 2|}{\sqrt{6} \cdot \sqrt{6}} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\text{23. (b): The equation of plane is } \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2x + 3y - 3z = 5$$

Distance of origin $(0, 0, 0)$ from the above plane is

$$\frac{\left| \frac{-5}{\sqrt{2^2 + 3^2 + 3^2}} \right|}{\sqrt{2^2 + 3^2 + 3^2}} = \frac{5}{\sqrt{22}}.$$

24. (d): Let $\vec{a}$ and $\vec{b}$ be two unit vectors

$$\Rightarrow |\vec{a} + \vec{b}|^2 = 1 \Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} = 1 \Rightarrow \vec{a} \cdot \vec{b} = -1/2 \Rightarrow \theta = 2\pi/3$$

25. (d)

26. (a): Since $|\vec{\alpha} - \vec{\beta}| < 1 \Rightarrow \alpha^2 + \beta^2 - 2\cos \phi < 1$

$$\Rightarrow 1 - \cos \phi < \frac{1}{2} \Rightarrow \cos \phi > \frac{1}{2} \therefore \phi \in \left[0, \frac{\pi}{3} \right)$$

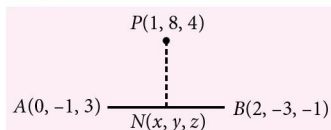
27. (b): d.r.'s of AB are $< 1, \lambda - 1, -2 >$

d.r.'s of CD are $< 1, 2, -2 >$ As $AB \parallel CD$

$$\therefore 1 = \frac{\lambda - 1}{2} = \frac{-2}{-2} \Rightarrow \lambda = 3$$

28. (c) : Equation of line AB is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = \lambda \text{ (say)}$$



Any point on AB is $(x, y, z) = (\lambda, -\lambda - 1, -2\lambda + 3)$

Now, d.r.s of PN are $\langle \lambda - 1, -(\lambda + 9), -(2\lambda + 1) \rangle$

As $PN \perp AB$

$$\therefore 1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$$

$$\Rightarrow \lambda = -\frac{5}{3} \therefore (x, y, z) = \left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$$

29. (b) : Let $a, b, c >$ are d.r.s of the line and it passes through $(3, 4, 5)$, so equation of line be

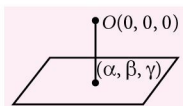
$$\frac{x-3}{a} = \frac{y-4}{b} = \frac{z-5}{c}$$

But, line is perpendicular to the plane which means normal to the plane is parallel to the line therefore

$$a = 2, b = 3, c = 1.$$

So, required equation of line is

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{1}$$



30. (b) : Given plane passes through $\vec{c}$ and is parallel to the vector $\vec{b} - \vec{c}$ and $\vec{a}$ so it is normal to $(\vec{b} - \vec{c}) \times \vec{a}$.

$$\text{Hence its equation is } (\vec{r} - \vec{c}) \cdot [(\vec{b} - \vec{c}) \times \vec{a}] = 0$$

$$\text{i.e., } \vec{r} \cdot (\vec{b} \times \vec{a} + \vec{a} \times \vec{c}) = [\vec{c} \vec{b} \vec{a}] \quad \dots(i)$$

$\therefore$ Length of perpendicular from $O(0, 0, 0)$ to (i) is

$$\frac{|\vec{O} \cdot (\vec{b} \times \vec{a} + \vec{a} \times \vec{c}) - [\vec{c} \vec{b} \vec{a}]|}{|\vec{b} \times \vec{a} + \vec{a} \times \vec{c}|} = \frac{[\vec{c} \vec{b} \vec{a}]}{|\vec{b} \times \vec{a} + \vec{a} \times \vec{c}|}$$

31. (d) : Required equation in vector form is

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$$

32. (c) : Given, intercepts $(a, b, c) = (1, 2, 3)$

Equation of plane in intercept form is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \Rightarrow \frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1 \Rightarrow 6x + 3y + 2z = 6$$

33. (c) : Let θ be the angle between the given line and plane.

$$\therefore \sin \theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}| |\vec{n}|}, \text{ Here } \vec{b} = 2\hat{i} - 3\hat{j} + 2\hat{k}, \vec{n} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

$$\therefore \sin \theta = \frac{6+6+10}{\sqrt{17}\sqrt{38}} \Rightarrow \theta = \sin^{-1} \left(\frac{22}{\sqrt{646}} \right)$$

34. (d)

$$35. (a) : \text{Required length} = \left| \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1^2 + 2^2 + (-3)^2}} \right|$$

$$\left| \frac{3-2+6}{\sqrt{1+4+9}} \right| = \frac{7}{\sqrt{14}}$$

$$\text{Also, vector projection of } \vec{a} \text{ on } \vec{b} = (\vec{a} \cdot \hat{b}) = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right)$$

Hence, both statements are true and Statement-II is the correct explanation of Statement-I.

36. (d) : $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 400, |\vec{a}| = 4$

$$\text{We know that, } (\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$\Rightarrow 400 = (4)^2 |\vec{b}|^2 \Rightarrow 16|\vec{b}|^2 = 400$$

$$\Rightarrow |\vec{b}|^2 = 25 \Rightarrow |\vec{b}| = 5$$

Hence, Statement-I is false.

$$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$$

$$= (\vec{a} \vec{b} \sin \theta)^2 + (\vec{a} \vec{b} \cos \theta)^2 = \vec{a}^2 \vec{b}^2$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$$

Hence, Statement-II is true.

37. (c) : In Statement-I the given cartesian equation is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$$

$$\Rightarrow \vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k} \quad \vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$$

The vector equation of the line is given by

$$\vec{r} = \vec{a} + \lambda \vec{b}, \lambda \in R \Rightarrow \vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

Thus Statement-I is true.

In Statement-II, it is given that the line passes through the

$$\text{point } (-2, 4, -5) \text{ and is parallel to } \frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$

Clearly, the direction ratios of line are $(3, 5, 6)$.

Now the equation of the line (in cartesian form) is

$$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6} \Rightarrow \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Hence, Statement-II is false.

38. (a) : Lines L_1 and L_2 are parallel to the vectors

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ respectively.}$$

The unit vector perpendicular to both L_1 and L_2 is

$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1+49+25}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

Now, equation of plane through $(-1, -2, -1)$ is

$$-(x+1) - 7(y+2) + 5(z+1) = 0 \text{ whose distance from}$$

$$(1, 1, 1) \text{ is } \frac{13}{5\sqrt{3}}.$$

39. (b) : P. We have, $|\vec{a} + \vec{b}| = |\vec{a} + 2\vec{b}|$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a} \cdot \vec{b}$
 $\Rightarrow 2\vec{a} \cdot \vec{b} = -3|\vec{b}|^2 < 0$

Hence, angle between $\vec{a}$ and $\vec{b}$ is obtuse.

Q. We have, $|\vec{a} + \vec{b}| = |\vec{a} - 2\vec{b}|$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + 4|\vec{b}|^2 - 4\vec{a} \cdot \vec{b}$
 $\Rightarrow 6\vec{a} \cdot \vec{b} = 3|\vec{b}|^2 > 0$

Hence, angle between $\vec{a}$ and $\vec{b}$ is acute.

R. We have, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$
 $\Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

S. $\vec{c} \times (\vec{a} \times \vec{b})$ lies in the plane of vector $\vec{a}$ and $\vec{b}$. A vector perpendicular to this plane is parallel to $\vec{a} \times \vec{b}$. Hence, angle is 0° .

40. (c) : P. Equation of the plane π is $p(x-1) + q(y-1) + r(z-1) = 0$ where $p+0 \cdot q-1 \cdot r=0$ and $-p+q+0 \cdot r=0$.
 $\Rightarrow p=q=r$ and the equation of π is $x+y+z=3$, which meets the co-ordinates axes at $A(3, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 3)$. So volume of the tetrahedron $= \frac{9}{2}$ cu units.

Q. $AB = BC = CA = 3\sqrt{2}$, so the area of the face ABC which is an equilateral triangle $= \frac{\sqrt{3}}{4} \times (3\sqrt{2})^2 = \frac{9\sqrt{3}}{2}$.

Each of the face OAB , OBC , OCA is an isosceles triangle with sides $3, 3, 3\sqrt{2}$.

So, area of each of these faces is

$$\frac{1}{2} \times 3\sqrt{2} \sqrt{9 - \frac{9}{2}} = \frac{9}{2} \text{ sq. units}$$

R. Required distance $= \left| \frac{\frac{3\sqrt{3}}{2} + 3\sqrt{3} + 3 - 3}{\sqrt{1+1+1}} \right| = \frac{9}{2} \text{ units}$

S. Distance of O from the face

$$ABC = \left| \frac{0+0+0-3}{\sqrt{1+1+1}} \right| = \sqrt{3}$$

Distance of A, B, C from opposite faces OBC, OCA, OAB are equal and is 3.

41. (b) 42. (c) 43. (a) 44. (d)
 45. (c) 46. (c) 47. (d) 48. (d)
 49. (b) 50. (b)



EXAM ALERT

MHT-CET 2023

Syllabus and Marking Scheme for
First Year of Engineering and Technology, Pharmacy Degree
Courses and
First Year of Master of Engineering and Technology
(Integrated), Pharm. D PG Degree course

- The questions will be based on Syllabus of State Council Of Educational Research And Training, Maharashtra.
- Approximately 20% weightage will be given to Std. XI curriculum and 80% weightage will be given to Std. XII curriculum while setting the question paper.
- There will be no Negative Marking, however difficulty level will be at par with the JEE (Main) for Mathematics, Physics, Chemistry and difficulty level for Biology will be at par with NEET. The questions will be mainly application based.
- MHT-CET will consist of 3 question papers of Multiple Choice Questions (MCQ) and each paper will be of 100 marks. Details are as given below :

Paper	Subject	No. of Multiple Choice Questions (MCQ) based on		Mark(s) Per Question	Total Marks	Duration in Minutes
		Std. XI	Std. XII			
Paper I	Mathematics	10	40	2	100	90
Paper II	Physics	10	40	1	100	90
	Chemistry	10	40			
Paper III	Biology	20	80	1	100	90

- The questions will be set on
 - Whole syllabus of Std XII of 2022-23 of Physics, Chemistry, Biology and Mathematics subjects

and

- Syllabus of Std XI of 2021-22 as mentioned below :

Sr.No.	Subject	Chapters/Units of Std XI Syllabus
1.	Physics	Motion in a plane, Laws of Motion, Gravitation, Thermal properties of matter, Sound, Optics, Electrostatics, Semiconductors
2.	Chemistry	Some Basic concepts of chemistry, Structure of atom, Chemical Bonding, Redox reactions, Elements of group 1 and 2, States of Matter(Gaseous and Liquids), Adsorption and colloids (Surface Chemistry), Hydrocarbons, Basic principles of organic chemistry
3.	Mathematics	Trigonometry II, Straight Line, Circle, Measures of Dispersion, Probability, Complex Numbers, Permutations and Combinations, Functions, Limits, Continuity
4.	Biology	Biomolecules, Respiration and Energy Transfer, Human Nutrition, Excretion and Osmoregulation.

Tentative Exam Dates

MHT CET - PCM	09-05-2023 to 13-05-2023
MHT CET - PCB	15-05-2023 to 20-05-2023

For more information visit cetcell.mahacet.org.

Are you ready for Olympiads?

LEVEL 2

Exam on 12th February 2023



SYLLABUS*

CLASS XI

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Sets, Relations and Functions, Principle of Mathematical Induction, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequalities, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Mathematical Reasoning, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Probability.

Practice Questions

- If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of A which contains atleast two elements is
A. 63 B. 57 C. 58 D. 64
- Let $f : (1, 3) \rightarrow R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is
A. $\left[\frac{2}{5}, \frac{3}{5}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right]$ B. $\left(\frac{2}{5}, \frac{4}{5}\right]$
C. $\left(\frac{3}{5}, \frac{4}{5}\right)$ D. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$
- If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, $(m, n \in N)$, then the greatest common divisor of the least values of m and n is
A. 1 B. 2 C. 4 D. 8
- For $x^2 - (a+3)|x| + 4 = 0$ to have real solutions, the range of a is
A. $(-\infty, -7] \cup [1, \infty)$ B. $(-3, \infty)$
C. $(-\infty, -7]$ D. $[1, \infty)$
- If $|x-1| + |x-3| \leq 8$, then the values of x lie in the interval
A. $(-\infty, -2]$ B. $[-2, 6]$
C. $(-3, 7)$ D. $(-2, \infty)$
- Four numbers whose sum is $\frac{65}{12}$ are in G.P. sum of their reciprocal is $\frac{65}{18}$ and product of first three of them is 1. If third term is α , then 2α equal to
A. 2 B. 3 C. $\frac{2}{3}$ D. $\frac{3}{2}$
- If $0 < x < \frac{\pi}{2}$, then $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$ is equal to
A. $2\sec x$ B. $2\cos x$ C. $\sec x$ D. $\cos x$
- The equation of one of the straight lines passing through the point $(0, 1)$ and is at a distance of $3/5$ units from the origin is
A. $4x + 3y = 3$ B. $-x + y = 1$
C. $x + y = 1$ D. $5x + 4y = 4$
- The hyperbola through foci of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and transverse and conjugate axes coincide with the major and minor axis of the ellipse. If product of eccentricities of ellipse and hyperbola is 1 then equation of hyperbola is
A. $x^2 - y^2 = 9$ B. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
C. $\frac{x^2}{9} - \frac{y^2}{25} = 1$ D. $\frac{x^2}{16} - \frac{y^2}{25} = 1$

10. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'Examination' is
 A. 2454 B. 2500
 C. 1600 D. None of these
11. The sum of the series $1 + \frac{1}{2} {}^nC_1 + \frac{1}{3} {}^nC_2 + \dots + \frac{1}{n+1} {}^nC_n$ is equal to
 A. $\frac{2^{n+1} - 1}{n+1}$ B. $\frac{3(2^n - 1)}{2n}$
 C. $\frac{2^n + 1}{n+1}$ D. $\frac{2^n + 1}{2n}$
12. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are
 A. 8, 13 B. 1, 20 C. 10, 11 D. 3, 18
13. If $I = \lim_{x \rightarrow 0} \sin \left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2} \right)$, then limit
 A. does not exist B. exists and equals 1
 C. exists and equals 0 D. exists and equals 1/2
14. If $y = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$, then $y'(x)$ is equal to
 A. $2\cos^2 x$ B. $2\cos^3 x$ C. $-\cos 2x$ D. $\cos 2x$
15. Which of the following is not a function?
 A. $\{(x, y) : x, y \in R, x^2 = y\}$
 B. $\{(x, y) : x, y \in R, y^2 = x\}$
 C. $\{(x, y) : x, y \in R, x = y^3\}$
 D. $\{(x, y) : x, y \in R, y = x^3\}$

ACHIEVERS SECTION

16. Match the following and select the correct option.

Column-I		Column-II	
(i)	A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a perfect square.	(p)	$\frac{1}{6}$
(ii)	Find the probability that in a random arrangement of letters of the word 'UNIVERSITY', two 'T's do not come together.	(q)	$\frac{1}{10}$
(iii)	If letters of the word 'PENCIL' are arranged in random order, what is the probability that N is always next to E?	(r)	$\frac{4}{5}$

- A. (i) $\rightarrow$ p, (ii) $\rightarrow$ r, (iii) $\rightarrow$ q
 B. (i) $\rightarrow$ q, (ii) $\rightarrow$ r, (iii) $\rightarrow$ p
 C. (i) $\rightarrow$ q, (ii) $\rightarrow$ p, (iii) $\rightarrow$ r
 D. (i) $\rightarrow$ p, (ii) $\rightarrow$ q, (iii) $\rightarrow$ r

17. Which one of the following is not correct for the features of exponential function given by $f(x) = b^x$ where $b > 1$?
 A. For very large negative values of x , the function is very close to 0.
 B. The domain of the function is R , the set of real numbers.
 C. The point (1, 0) is always on the graph of the function.
 D. The range of the function is the set of all positive real numbers.
18. Read the statements carefully and select the correct option.

Statement-1 : Slope of the line

$$(\cos \theta + \sin \theta)x + \sin 2\theta y = 1 \text{ is } \frac{\sqrt{3} - 1}{\sqrt{3}} \text{ if } \theta = \frac{\pi}{6}$$

Statement-2 : $\sqrt{3}x + y = 30$ represents the line whose perpendicular from origin makes an angle $\pi/6$ with the +ve direction of x -axis and is at a distance 15 units from the origin.

- A. Both Statement-1 and Statement-2 are true.
 B. Statement-1 is true but Statement-2 is false.
 C. Statement-1 is false but Statement-2 is true.
 D. Both Statement-1 and Statement-2 are false.

19. Read the statements carefully and state "T" for true and "F" for false.

(i) if $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then $B = C$

(ii) $(A' \cap B' \cap C) \cup (B \cap C) \cup (C \cap A) = B$

(iii) For any three sets A_1, A_2, A_3 , let $B_1 = A_1$, $B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$.

(i) (ii) (iii)

- A. T T F
 B. T F T
 C. T F F
 D. F F T

20. From a group of 6 men and 8 women a committee is formed, then number of committee in which at least 2 men are always selected and number of women is double the number of men is
 A. 1625 B. 1615 C. 1525 D. 1610

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (B) : We have, $n(A) = 6$

Total number of subsets of $A = 2^6$

$\therefore$ Number of subsets of A which contains atleast two elements $= 2^6 - 7$

[$\because$ Subsets of A having none or 1 element are $\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$]

$$= 64 - 7 = 57$$

$$2. (D) : f(x) = \begin{cases} \frac{x}{x^2 + 1}; & x \in (1, 2) \\ \frac{2x}{x^2 + 1}; & x \in [2, 3) \end{cases}$$

$\Rightarrow f(x)$ is a strictly decreasing function.

$$\therefore f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right]$$

$$\Rightarrow f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$$

$$3. (C) : \text{We have, } \left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$$

$$\Rightarrow \left[\frac{(1+i)^2}{2}\right]^{m/2} = \left[\frac{(1+i)^2}{-2}\right]^{n/3} = 1$$

$$\Rightarrow (i)^{m/2} = (-i)^{n/3} = (i)^4$$

$$\Rightarrow \frac{m}{2} = 4k_1 \text{ and } \frac{n}{3} = 4k_2 \Rightarrow m = 8k_1 \text{ and } n = 12k_2$$

So, least values of m and n are 8 and 12 respectively.

$$\therefore \text{GCD}(8, 12) = 4$$

4. (D) : Given, $x^2 - (a+3)|x| + 4 = 0$ have real solution.

$$\therefore (a+3)^2 - 4 \times 4 \geq 0$$

$$\Rightarrow a^2 + 6a - 7 \geq 0 \Rightarrow (a-1)(a+7) \geq 0$$

$$\Rightarrow a \in [1, \infty) \text{ and } a \in (-\infty, -7]$$

But for $a \in (-\infty, -7]$ the given question does not have any positive real solutions.

$$\therefore a \in [1, \infty)$$

5. (B) : Here two cases arises,

Case 1 : When $-\infty < x \leq 1$.

Then, $(x-1) \leq 0, x-3 < 0$

$$\Rightarrow |x-1| = -(x-1) \text{ and } |x-3| = -(x-3)$$

$$\text{Now, } |x-1| + |x-3| \leq 8 \Rightarrow -(x-1) - (x-3) \leq 8$$

$$\Rightarrow -2x + 4 \leq 8 \Rightarrow x \geq -2 \quad \dots(i)$$

Case 2 : When $3 \leq x < \infty$

$$\therefore |x-1| = (x-1) \text{ and } |x-3| = (x-3)$$

$$\text{Now, } |x-1| + |x-3| \leq 8$$

$$\Rightarrow x-1 + x-3 \leq 8 \Rightarrow 2x-4 \leq 8 \Rightarrow x \leq 6 \quad \dots(ii)$$

From (i) and (ii), we get $x \in [-2, 6]$

6. (B) : Let the four numbers be a, ar, ar^2, ar^3 , where a is the first term and r is the common ratio.

$$\therefore \text{We have, } a + ar + ar^2 + ar^3 = \frac{65}{12} \text{ (Given)}$$

$$\Rightarrow a(1 + r + r^2 + r^3) = \frac{65}{12} \quad \dots(i)$$

$$\text{Also, } \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18} \quad \text{(Given)}$$

$$\Rightarrow \frac{1}{a} \left(1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}\right) = \frac{65}{18}$$

$$\Rightarrow \frac{r^3 + r^2 + r + 1}{r^3} = \frac{65a}{18}$$

$$\Rightarrow \frac{65}{12ar^3} = \frac{65a}{18} \quad \text{(Using (i))}$$

$$\Rightarrow 12a^2r^3 = 18 \Rightarrow a^2r^3 = \frac{3}{2} \quad \dots(ii)$$

$$\text{Again, } a \cdot ar \cdot ar^2 = 1 \quad \text{(Given)}$$

$$\Rightarrow r^3 = \frac{1}{a^3} \quad \dots(iii)$$

Putting (iii) in (ii), we get

$$a^2 \cdot \frac{1}{a^3} = \frac{3}{2} \Rightarrow a = \frac{2}{3}$$

$$\text{From (iii), } r = \frac{3}{2}$$

$$\text{Now, } \alpha = \text{Third term} = ar^2 = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} = \frac{3}{2}$$

$$\text{Hence, } 2\alpha = 2 \times \frac{3}{2} = 3$$

$$7. (A) : \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} + \frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} = \frac{\left(1 + \tan \frac{x}{2}\right)^2 + \left(1 - \tan \frac{x}{2}\right)^2}{1 - \tan^2 \frac{x}{2}}$$

$$= \frac{1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} + 1 + \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = 2 \sec x$$

8. (A) : Equation of line passes through the point (0, 1) is $y - 1 = m(x - 0)$
 $\Rightarrow mx - y + 1 = 0$... (i)

Now, distance of line (i) from origin is $\frac{3}{5}$ units

$$\therefore \frac{0 + 0 + 1}{\sqrt{m^2 + 1}} = \pm \frac{3}{5} \Rightarrow m^2 + 1 = \frac{25}{9} \Rightarrow m = \pm \frac{4}{3}$$

So, the equation of lines are

$$\frac{4}{3}x - y + 1 = 0, \frac{-4}{3}x - y + 1 = 0$$

$$\Rightarrow 4x - 3y + 3 = 0, 4x + 3y - 3 = 0$$

9. (B) : Let e be the eccentricity of given ellipse.

$$\therefore e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - \frac{16}{25} \Rightarrow e = \frac{3}{5}$$
 ... (i)

Now, let e_n be the eccentricity of required hyperbola.

$$\therefore e \times e_n = 1 \quad (\text{Given})$$

$$\Rightarrow e_n = \frac{5}{3} \quad (\text{Using (i)})$$
 ... (ii)

We know that, foci of ellipse are $(\pm ae, 0)$, so foci of given ellipse $\equiv (\pm 3, 0)$

Let the equation of required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
 ... (iii)

Since, this passes through $(\pm 3, 0)$.

$$\therefore \frac{9}{a^2} - 0 = 1 \Rightarrow a^2 = 9$$
 ... (iv)

$$\text{Also, } e_n^2 = 1 + \frac{b^2}{a^2}$$

By using (ii) and (iv), we get $b^2 = 16$

$$\text{Thus, (iii) becomes } \frac{x^2}{9} - \frac{y^2}{16} = 1$$

10. (A) : The word EXAMINATION has 2N, 2A, 2I, E, X, M, T, O

Case I : If all letters are different, then number of words

$$= {}^8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

Case II : If 2 letters are same and 2 are different, then number of words = ${}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!} = 3 \cdot 21 \cdot 12 = 756$

Case III : If 2 letters are same of one kind and 2 letters are same of other kind, then number of words

$$= {}^3C_2 \cdot \frac{4!}{2! \cdot 2!} = 3 \cdot 6 = 18$$

$$\therefore \text{Total number of words} = 1680 + 756 + 18 = 2454$$

$$11. (A) : 1 + \frac{1}{2} \cdot n + \frac{1}{3} \cdot \frac{n(n-1)}{2!} + \dots + \frac{1}{n+1}$$

$$= \frac{1}{n+1} \left[(n+1) + \frac{(n+1)n}{2!} + \frac{(n+1)n(n-1)}{3!} + \dots + 1 \right]$$

$$= \frac{1}{n+1} [{}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \dots + {}^{n+1}C_{n+1}]$$

$$= \frac{1}{n+1} [{}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + \dots + {}^{n+1}C_{n+1} - 1]$$

$$[\because {}^{n+1}C_0 = 1]$$

$$= \frac{1}{n+1} [2^{n+1} - 1]$$

12. (C) : Let the remaining two observations be a and

$$b. \text{ Then, mean} = \frac{2 + 4 + 5 + 7 + a + b}{6} = 6.5 \quad (\text{Given})$$

$$\Rightarrow a + b = 21$$
 ... (i)

Also, variance = 10.25 (Given)

$$\Rightarrow \frac{1}{6}(2^2 + 4^2 + 5^2 + 7^2 + a^2 + b^2) - (6.5)^2 = 10.25$$

$$\Rightarrow a^2 + b^2 = 315 - 94 = 221$$
 ... (ii)

$$\text{Now, } (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$\Rightarrow (a-b)^2 = 2 \times 221 - (21)^2 \quad [\text{From (i) and (ii)}]$$



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$$\Rightarrow (a-b)^2 = 1 \Rightarrow a-b = \pm 1$$

If $a-b = 1$, then $a = 11$, $b = 10$

If $a-b = -1$, then $a = 10$, $b = 11$

$$\begin{aligned} 13. (C): I &= \lim_{x \rightarrow 0} \sin \left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \sin \left[\frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right) - x - 1 - \frac{x^2}{2}}{x^2} \right] \\ &= \lim_{x \rightarrow 0} \sin \left(\frac{\frac{x^3}{3!} + \frac{x^4}{4!} + \dots}{x^2} \right) = \lim_{x \rightarrow 0} \sin \left[\frac{x}{3!} + \frac{x^2}{4!} + \dots \right] = 0 \end{aligned}$$

Thus, limit exists and equals to 0.

$$\begin{aligned} 14. (C): \text{ We have, } y &= \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x} \\ &= \frac{\sin^2 x \cdot \tan x}{1 + \tan x} + \frac{\cos^2 x}{1 + \tan x} = \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} \\ &= \frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)}{(\sin x + \cos x)} \end{aligned}$$

$$\Rightarrow y = 1 - \sin x \cos x$$

$$\therefore \frac{dy}{dx} = -[\sin x(-\sin x) + \cos x \cdot \cos x] = -\cos 2x$$

15. (B)

16. (B) : (i) A : Getting a number which is a perfect square

$$A = (1, 4, 9, 16, 25, 36, 49, 64, 81, 100)$$

$$n(A) = 10, n(S) = 100$$

$$\therefore \text{ Required probability} = P(A) = \frac{n(A)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(ii) Out of the letters in the word 'UNIVERSITY' two letter 'I' are alike.

$$\therefore \text{ Number of permutations} = \frac{10!}{2}$$

Number of words in which two 'I' are never together
= Total number of words - Number of words in which two 'I' are together

$$= \frac{10!}{2} - 9! = 4 \cdot 9!$$

$$\therefore \text{ Required probability} = \frac{9! \cdot 4}{10! / 2} = \frac{8}{10} = \frac{4}{5}$$

(iii) Number of ways in which 6 letters of the word 'PENCIL' can be arranged is 6!

If N is next to E, then they can be arranged in 5! ways.

$$\therefore \text{ Required probability} = \frac{5!}{6!} = \frac{1}{6}$$

17. (C)

18. (C) : Slope of line in Statement-1 is

$$\frac{-(\cos \theta + \sin \theta)}{2 \sin \theta \cos \theta} = \frac{-\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)}{2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}} = \frac{-(\sqrt{3} + 1)}{\sqrt{3}} \text{ for } \theta = \pi/6$$

So Statement-1 is false.

In Statement-2, given equation can be written as

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 15$$

$$\text{or } x \cos \pi/6 + y \sin \pi/6 = 15$$

$$\text{or } x \cos \alpha + y \sin \alpha = p, \text{ where } \alpha = \pi/6, p = 15$$

So Statement-2 is true.

19. (B) : (i) We have, $B = B \cup (A \cap B)$

$$= B \cup (A \cap C) \quad [\because A \cap B = A \cap C]$$

$$= (B \cup A) \cap (B \cup C)$$

$$= (A \cup C) \cap (B \cup C) \quad [\because (A \cup C) = (A \cup B)]$$

$$= (A \cap B) \cup C = (A \cap C) \cup C = C$$

Hence $B = C$.

$$(ii) (A' \cap B' \cap C) \cup (B \cap C) \cup (C \cap A)$$

$$= [(A \cup B)' \cap C] \cup [(B \cup A) \cap C]$$

$$= [C \cap (A \cup B)'] \cup [C \cap (A \cup B)]$$

$$= C \cap [(A \cup B)' \cup (A \cup B)] = C \cap U = C$$

$$(iii) B_1 \cup B_2 = A_1 \cup (A_2 - A_1) = A_1 \cup (A_2 \cap A_1')$$

$$= (A_1 \cup A_2) \cap (A_1 \cup A_1') = (A_1 \cup A_2) \cap U = A_1 \cup A_2$$

$$B_1 \cup B_2 \cup B_3 = (B_1 \cup B_2) \cup B_3 = (A_1 \cup A_2) \cup [A_3 - (A_1 \cup A_2)]$$

$$= (A_1 \cup A_2) \cup [A_3 \cap (A_1 \cup A_2)']$$

$$= [(A_1 \cup A_2) \cup A_3] \cap [(A_1 \cup A_2) \cup ((A_1 \cup A_2)')]]$$

$$= [A_1 \cup A_2 \cup A_3] \cap U = A_1 \cup A_2 \cup A_3$$

20. (A) : We have, total men = 6 and total women = 8

$\therefore$ The required number of ways

$$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$$

$$= 15 \times 70 + 20 \times 28 + 15 \times 1 = 1625$$

For other sections/subjects please refer to
Physics For You, Chemistry Today and Biology Today





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LEVEL 2

Exam on 12th February 2023



CLASS XII

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

Practice Questions

- Let the relation ρ be defined on R by $a \rho b$ holds if and only if $a - b$ is zero or irrational, then
 - ρ is equivalence relation.
 - ρ is reflexive and symmetric but is not transitive.
 - ρ is reflexive and transitive but is not symmetric.
 - ρ is reflexive only.
- Let $A = \{x \in R : -1 \leq x \leq 1\}$ and $f : A \rightarrow A$ be a mapping defined by $f(x) = x|x|$. Then f is
 - injective but not surjective
 - surjective but not injective
 - neither injective nor surjective
 - bijective
- The domain of the function $f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$ is
 - $[0, 2]$
 - $[0, 2)$
 - $[1, 2)$
 - $[1, 2]$
- Let A and B be two square matrices of order 3 and $AB = O_3$, where O_3 denotes the null matrix of order 3. Then,
 - must be $A = O_3, B = O_3$
 - if $A \neq O_3$, must be $B \neq O_3$
 - if $A = O_3$, must be $B \neq O_3$
 - may be $A \neq O_3, B \neq O_3$
- If α, β are non-real numbers satisfying $x^3 - 1 = 0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to
 - 0
 - λ^3
 - $\lambda^3 + 1$
 - None of these
- The function $f(x) = x - |x - x^2|$, $-1 \leq x \leq 1$ is continuous on the interval
 - $[-1, 1]$
 - $(-1, 1)$
 - $[-1, 1] \setminus \{0\}$
 - $(-1, 1) \setminus \{0\}$
- If $f(x) = \begin{cases} x^p \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at $x = 0$, $f(x)$ is
 - continuous if $p > 0$ and differentiable if $p > 1$
 - continuous if $p > 1$ and differentiable if $p > 2$
 - continuous and differentiable if $p > 0$
 - none of these
- The function f given by $f(x) = (x^2 - 3)e^x$ is decreasing on the interval
 - $(-3, \infty)$
 - $(1, \infty)$
 - $(-\infty, 1)$
 - $(-3, 1)$

9. If $\int \frac{\sin 2x}{(a+b \cos x)^2} dx = \alpha \left[\log_e |a+b \cos x| + \frac{a}{a+b \cos x} \right] + c$, then $\alpha =$

A. $\frac{2}{b^2}$ B. $\frac{2}{a^2}$ C. $-\frac{2}{b^2}$ D. $-\frac{2}{a^2}$

10. The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ is
A. $1/2$ B. 2 C. 3 D. $9/2$

11. The solution curve of the differential equation, $(1+e^{-x})(1+y^2) \frac{dy}{dx} = y^2$, which passes through the point $(0, 1)$, is

A. $y^2 + 1 = y \left(\log_e \left(\frac{1+e^{-x}}{2} \right) + 2 \right)$
B. $y^2 + 1 = y \left(\log_e \left(\frac{1+e^x}{2} \right) + 2 \right)$
C. $y^2 = 1 + y \log_e \left(\frac{1+e^x}{2} \right)$
D. $y^2 = 1 + y \log_e \left(\frac{1+e^{-x}}{2} \right)$

12. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$, then the area of parallelogram having diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is

A. $4\sqrt{6}$ sq. units B. $\frac{1}{2}\sqrt{21}$ sq. units
C. $\frac{\sqrt{6}}{2}$ sq. units D. $\sqrt{6}$ sq. units

13. If the shortest distance between the lines $\vec{r}_1 = \alpha \hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in R$, $\alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in R$ is 9, then α is
A. 27 B. 6 C. 12 D. 3

14. Consider a Linear Programming Problem :
Minimize $Z = 5x + 3y$,
Subject to : $3x + y \geq 10$, $2x + 2y \geq 14$ and $x + 2y \geq 9$.
Which one of the following points lies outside the feasible region?

A. $(1, 9)$ B. $(4, 2)$ C. $(6, 2)$ D. $(12, 2)$

15. Events E_1 and E_2 form a partition of the sample space S . A is any event such that $P(E_1) = P(E_2) = 1/2$, $P(E_2|A) = \frac{1}{2}$ and $P(A|E_2) = \frac{2}{3}$, then $P(E_1|A)$ is
A. $2/3$ B. 1 C. $1/4$ D. $1/2$

ACHIEVERS SECTION

16. Read the given statement carefully and state 'T' for true and 'F' for false.

- (i) If A is a square matrix such that $A^2 = A$ then $(I + A)^3 - 7A = O$.
(ii) Let B and C be two square matrices such that $BC = CB$ and $C^2 = O$. If $A = B + C$ then $A^3 - B^3 - 3B^2C = O$.
(iii) $B'AB$ is symmetric if B is square matrix and A is symmetric matrix.

	(i)	(ii)	(iii)
A.	T	F	T
B.	F	T	T
C.	T	T	F
D.	F	F	T

17. Fill in the blanks and select the correct option.

- (i) If $A = \begin{bmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{bmatrix}$ and let $|A| = 256$, then $|k| =$

P

- (ii) If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, then $\text{adj } A =$ **Q**.

- (iii) If $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then A^{-1} is **R**.

	(i)	(ii)	(iii)
A.	8	A	A
B.	-8	A^T	A
C.	16	$-A$	A^T
D.	8	A	A^T

18. Read the following statements carefully and select and correct option.

Statement-1 : $\int \frac{d(x^2 + 1)}{\sqrt{x^2 + 2}}$ is equal to $2\sqrt{x^2 + 2} + C$.

Statement-2 :

$\int \frac{x^{9/2}}{\sqrt{1+x^{11}}} dx$ is $\frac{2}{11} \log |x + \sqrt{1+x^{11}}| + C$.

- A. Both Statement-1 and Statement-2 are true.
B. Both Statement-1 and Statement-2 are false.
C. Statement-1 is true but Statement-2 is false.
D. Statement-1 is false but Statement-2 is true.

19. Match the following and select the correct option.

Column - I		Column - II	
(i)	The solution of $\frac{dy}{dx} = 2^{y-x}$ is	(p)	$x^2 (y+3)^3 = e^{y+2}$
(ii)	The solution of $\frac{dy}{dx} = 1+x+y^2+xy^2$, when $y=0, x=0$ is	(q)	$2^{-x} - 2^{-y} = k$
(iii)	Solution of $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is	(r)	$y = \tan\left(x + \frac{x^2}{2}\right)$

(iv)	The solution of $2(y+3) - xy \frac{dy}{dx} = 0$, given that $y(1) = -2$ is	(s)	$\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$
------	---	-----	---

- A. (i) $\rightarrow$ (q); (ii) $\rightarrow$ (r); (iii) $\rightarrow$ (p); (iv) $\rightarrow$ (s)
 B. (i) $\rightarrow$ (r); (ii) $\rightarrow$ (q); (iii) $\rightarrow$ (s); (iv) $\rightarrow$ (p)
 C. (i) $\rightarrow$ (r); (ii) $\rightarrow$ (q); (iii) $\rightarrow$ (p); (iv) $\rightarrow$ (s)
 D. (i) $\rightarrow$ (q); (ii) $\rightarrow$ (r); (iii) $\rightarrow$ (s); (iv) $\rightarrow$ (p)

20. Area bounded by the curve $y = \log_e (\sin x)$ and x -axis for $x \in (0, \pi)$ is

- A. $\pi \log_e 2$ sq. units B. $2\pi \log_e 2$ sq. units
 C. $\frac{\pi}{2} \log_e 2$ sq. units D. None of these

Darken your choice with HB Pencil

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (B) : Relation ρ is defined as

$a \rho b$ iff $a - b$ is zero or irrational.

(i) $a \rho a = 0 \Rightarrow a - a = 0$

$\therefore \rho$ is reflexive.

(ii) $a \rho b \Rightarrow a - b$ is zero or irrational.

$\Rightarrow b - a$ is zero or irrational.

$\Rightarrow b \rho a$

$\therefore \rho$ is symmetric.

(iii) $a \rho b$ and $b \rho c \Rightarrow a - b$ is zero or irrational and $b - c$ is zero or irrational

But $a - c$ is neither zero nor irrational when $a = 2$, $b = \sqrt{3}$ and $c = 3$.

$\therefore \rho$ is not transitive.

2. (D) : We have, $f(x) = \begin{cases} x^2 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

Clearly, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one.

Since, $0 < x \leq 1 \Rightarrow 0 < x^2 \leq 1 \Rightarrow 0 < f(x) \leq 1$

$-1 \leq x < 0 \Rightarrow 0 < x^2 \leq 1 \Rightarrow -1 \leq -x^2 < 0$

$\Rightarrow -1 \leq f(x) < 0$;

and $x = 0 \Rightarrow f(x) = 0$

$\therefore \text{Range}(f) = [-1, 1] = A$

Thus, $f(x)$ is onto

Hence, $f(x)$ is bijective.

3. (C) : Given, $f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$

$\sqrt{4-x^2}$ is defined for $4-x^2 \geq 0$

$\Rightarrow x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$

and $\sin^{-1}(2-x)$ is defined for $-1 \leq 2-x \leq 1$

$\Rightarrow -3 \leq -x \leq -1 \Rightarrow 1 \leq x \leq 3$

Also, $\sin^{-1}(2-x) = 0$ for $x = 2$

$\therefore \text{Domain of } f(x) = [-2, 2] \cap [1, 3] - \{2\} = [1, 2)$

4. (D) : Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

be two square matrices of order 3 such that

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = O_3 \text{ but } A \neq O_3 \text{ and } B \neq O_3.$$

5. (B) : We have, $x^3 - 1 = 0$

Since, α and β are its non-real roots

i.e., 1, α and β are the roots of $x^3 - 1 = 0$

$\Rightarrow 1 + \alpha + \beta = 0$ and $1 \times \alpha\beta = 1 \Rightarrow \alpha + \beta = -1$ and $\alpha\beta = 1$

Also, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-1)^2 - 2 \times 1 = -1$
 ... (i)

Now,
$$\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$$

$$= (\lambda+1)[\lambda^2 + (\alpha+\beta)\lambda + \alpha\beta - 1] - \alpha[\alpha\lambda + \alpha^2 - \beta] + \beta[\alpha - \beta\lambda - \beta^2]$$

$$= (\lambda+1)[\lambda^2 + (-1)\lambda + 1 - 1] - \alpha^2\lambda - \alpha^3 + \alpha\beta + \beta\alpha - \beta^2\lambda - \beta^3$$

$$= \lambda^3 - \lambda - (\alpha^2 + \beta^2)\lambda - \alpha^3 - \beta^3 + 2\alpha\beta$$

$$= \lambda^3 - \lambda - (-1)\lambda - 1 - 1 + 2 \times 1$$

[$\because \alpha$ and β are roots of $x^3 - 1 = 0$ i.e., $\alpha^3 = 1$ and $\beta^3 = 1$]

$$= \lambda^3 - \lambda + \lambda - 1 - 1 + 2 = \lambda^3$$

6. (A) : Given, $f(x) = x - |x - x^2|$, $-1 \leq x \leq 1$

$$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \leq x \leq 0 \\ x^2, & 0 \leq x \leq 1 \end{cases}$$

Now, $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x - x^2) = 0$

and $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0$

Also, $f(0) = 0$

$\therefore f(x)$ is continuous in the interval $[-1, 1]$.

7. (A) : We have, $f(0) = 0$

Now, L.H.L. = $\lim_{x \rightarrow 0^-} x^p \cos \frac{1}{x} = \lim_{h \rightarrow 0} (-h)^p \cos \left(\frac{1}{-h} \right)$

and R.H.L. = $\lim_{x \rightarrow 0^+} x^p \cos \frac{1}{x} = \lim_{h \rightarrow 0} h^p \cos \left(\frac{1}{h} \right)$

$\therefore$ For the function to be continuous at $x = 0$, the L.H.L. and R.H.L. should be equal to zero.

$\therefore p > 0$ (As $\cos \frac{1}{x}$ is a bounded function)

Now, $Rf'(0) = \lim_{x \rightarrow 0} \frac{h^p \cos \left(\frac{1}{h} \right) - 0}{h} = \lim_{h \rightarrow 0} h^{p-1} \cos \left(\frac{1}{h} \right)$

Similarly, $Lf'(0) = \lim_{h \rightarrow 0} (-h)^{p-1} \cos \left(\frac{1}{-h} \right)$

Thus, the function to be differentiable at $x = 0$, the limit value should be equal to zero

$\therefore p - 1 > 0$ i.e., $p > 1$

8. (D) : We have, $f(x) = (x^2 - 3)e^x$

Differentiating both sides, we get

$$f'(x) = (x^2 - 3) \cdot e^x + e^x \cdot 2x = e^x(x^2 + 2x - 3)$$

$$= e^x(x+3)(x-1)$$

For $f(x)$ to be decreasing, we should have $f'(x) \leq 0$

$$\Rightarrow e^x(x+3)(x-1) \leq 0$$

Sign of $f'(x)$ $\xleftarrow{+ve} \begin{matrix} -\infty & -3 & 1 & \infty \end{matrix} \xrightarrow{+ve}$

$\therefore f(x)$ is decreasing in the interval $(-3, 1)$.

9. (C) : Let $I = \int \frac{\sin 2x}{(a+b \cos x)^2} dx$

$$\Rightarrow I = \int \frac{2 \sin x \cos x}{(a+b \cos x)^2} dx$$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$$\therefore I = -2 \int \frac{t dt}{(a+bt)^2}$$

Now, let $t = A(a+bt) + B \frac{d}{dt}(a+bt)$

$$\Rightarrow t = Aa + bAt + Bb$$

$$\Rightarrow Aa + Bb = 0 \text{ and } bA = 1 \Rightarrow A = 1/b$$

$$\Rightarrow B = \frac{-Aa}{b} = \frac{-a}{b^2}$$

$$\therefore I = -2 \int \left[\frac{\frac{1}{b}(a+bt) - \frac{a}{b^2}(b)}{(a+bt)^2} \right] dt$$

$$= \frac{-2}{b} \int \frac{1}{a+bt} dt + \frac{2a}{b} \int \frac{1}{(a+bt)^2} dt$$

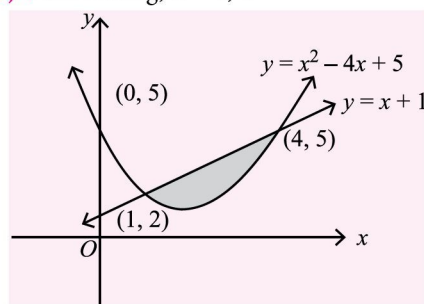
$$= \frac{-2}{b} \frac{\log_e |a+bt|}{b} + \frac{2a}{b} \left(\frac{-1}{a+bt} \right) \frac{1}{b} + c$$

$$= \frac{-2}{b^2} \log_e |a+b \cos x| - \frac{2a}{b^2} \left(\frac{1}{a+b \cos x} \right) + c$$

$$= \alpha \left[\log_e |a+b \cos x| + \frac{a}{a+b \cos x} \right] + c \quad [\text{Given}]$$

$$\therefore \alpha = \frac{-2}{b^2}$$

10. (D) : On solving, $x = 1, 4$



$$\therefore \text{Required area} = \int_1^4 [(x+1) - (x^2 - 4x + 5)] dx$$

$$= \int_1^4 (5x - x^2 - 4) dx = \left[\frac{5x^2}{2} - \frac{x^3}{3} - 4x \right]_1^4 = \frac{9}{2} \text{ sq. units}$$

11. (C) : The given differential equation is,

$$(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2 \Rightarrow \int \frac{1+y^2}{y^2} dy = \int \frac{dx}{1+e^{-x}}$$

$$\Rightarrow \int \frac{1}{y^2} dy + \int dy = \int \frac{e^x}{e^x+1} dx$$

$$\Rightarrow \frac{-1}{y} + y = \log(e^x+1) + C \quad \dots(i)$$

Now, at $(0, 1)$, (i) becomes, $0 = \log 2 + C \Rightarrow C = -\log 2$
So, the required solution is

$$y^2 - 1 = y \left\{ \log \frac{(e^x+1)}{2} \right\} \Rightarrow y^2 = 1 + y \left\{ \log \frac{(e^x+1)}{2} \right\}$$

12. (A) : Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

and $\vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$

Let $\vec{A} = \vec{a} + \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 3\hat{j} + 5\hat{k})$

$= (2\hat{i} + 4\hat{j} + 6\hat{k})$ and $\vec{B} = \vec{b} + \vec{c}$

$= (\hat{i} + 3\hat{j} + 5\hat{k}) + (7\hat{i} + 9\hat{j} + 11\hat{k}) = 8\hat{i} + 12\hat{j} + 16\hat{k}$

Since $\vec{A}$ and $\vec{B}$ are diagonals.

$$\therefore \text{Area of parallelogram} = \frac{1}{2} |\vec{A} \times \vec{B}|$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix} = \frac{1}{2} [\hat{i}(64-72) - \hat{j}(32-48) + \hat{k}(24-32)]$$

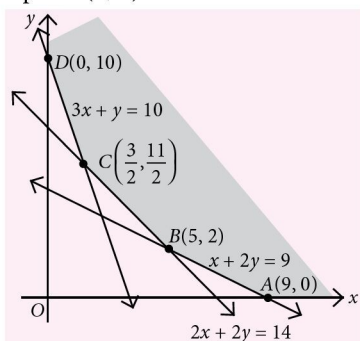
$$= \frac{1}{2} |-8\hat{i} + 16\hat{j} - 8\hat{k}| = \sqrt{(-4)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{96} = 4\sqrt{6} \text{ sq. units}$$

13. (B)

14. (B) : On plotting the constraints, we get $A(9, 0)$, $B(5, 2)$, $C\left(\frac{3}{2}, \frac{11}{2}\right)$ and $D(0, 10)$ as corner points of feasible region which is unbounded.

Now, $(1, 9)$, $(6, 2)$ and $(12, 2)$ lies inside the feasible region and point $(4, 2)$ lies outside the feasible region.



15. (D) : We have, $P(E_1) = P(E_2) = \frac{1}{2}$

$P(E_2/A) = \frac{1}{2}$, $P(A/E_2) = \frac{2}{3}$. Let $P\left(\frac{A}{E_1}\right)$ be p .

Using Bayes' theorem, we have

$$P(E_2/A) = \frac{P(A/E_2) \cdot P(E_2)}{P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2)}$$

$$\Rightarrow \frac{1}{2} = \frac{\frac{2}{3} \cdot \frac{1}{2}}{p \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2}} \Rightarrow p + \frac{2}{3} = \frac{2}{3} \times 2$$

$$\Rightarrow p = \frac{2}{3} \Rightarrow P(A/E_1) = \frac{2}{3}$$

$$\text{Now, } P(E_1/A) = \frac{P(A/E_1) \cdot P(E_1)}{P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2)}$$

$$= \frac{\frac{2}{3} \cdot \frac{1}{2}}{\frac{2}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2}} = \frac{1}{2}$$

16. (B) : (i) If $A^2 = A, (I + A)^3 = I^3 + 3A + 3A^2 + A^3$
 $= I + 7A$

(ii) If $A = B + C, A^3 = (B + C)^3 = B^3 + 3B^2C + 3BC^2 + C^3$
 $\Rightarrow A^3 - B^3 - 3B^2C = 3BC^2 + C^3 = (3B + C)C^2 = O$

(iii) Let A be a symmetric matrix. Then $A' = A$.

Now, $(B' A B)' = B' A' (B') = B' A' B$

$$[\because (AB)' = B' A' \text{ and } (B')' = B]$$

$$= B' A B$$

$\therefore B' A B$ is symmetric matrix.

mtg

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17. (D) : (i) We have, $A = \begin{vmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{vmatrix}$

Now, expanding along C_1 , we get $|A| = 4(k^2)$

But $\det(A) = 256$ (Given)

On comparing, we get

$$4k^2 = 256 \Rightarrow k^2 = 64 \Rightarrow k = \pm 8. \text{ Hence, } |k| = 8$$

(ii) The matrix of cofactors is $\begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}$

$$\therefore \text{adj } A = A.$$

(iii) We have, $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$$\text{Hence, } \text{adj}(A) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } |A| = 0 + (1)(-1) = -1$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A^T$$

18. (C) : Let $I = \int \frac{dt}{\sqrt{1+t}}$, where $x^2 + 1 = t$

$$= 2\sqrt{1+t} + C = 2\sqrt{x^2+1} + C$$

$\therefore$ Statement-1 is true.

$$\text{Let } I_1 = \int \frac{x^{9/2}}{\sqrt{1+x^{11}}} dx$$

$$\text{Putting } x^{11/2} = t \Rightarrow \frac{11}{2} x^{9/2} dx = dt$$

$$\Rightarrow I_1 = \frac{2}{11} \log(x^{11/2} + \sqrt{1+x^{11}}) + C$$

$\therefore$ Statement-2 is false.

19. (D) : (i) We have, $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

$$\text{Integrating both sides, we get } \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$$\Rightarrow -2^{-y} + 2^{-x} = C \log 2 = k(\text{say}) \Rightarrow 2^{-x} - 2^{-y} = k$$

(ii) We have, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$$\Rightarrow \frac{dy}{dx} = (1+y^2)(1+x) \Rightarrow \frac{dy}{1+y^2} = (1+x)dx$$

$$\text{Integrating both sides, we get } \tan^{-1} y = x + \frac{x^2}{2} + C \quad \dots(i)$$

$$\text{When, } y = 0, x = 0; \tan^{-1}(0) = 0 + 0 + C \therefore C = 0$$

$$\text{Now, from (i), } \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

(iii) We have, $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y^2}{x^2}\right) \quad \dots(ii)$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\text{From (ii), we get } v + x \frac{dv}{dx} = 1 + v + v^2$$

$$\Rightarrow x \frac{dv}{dx} = 1 + v^2 \Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$$

Integrating both sides, we get

$$\tan^{-1} v = \log |x| + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log |x| + C$$

(iv) We have, $2(y+3) - xy \frac{dy}{dx} = 0$

$$\Rightarrow 2(y+3) = xy \frac{dy}{dx} \Rightarrow 2 \frac{dx}{x} = \left(\frac{y}{y+3}\right) dy$$

$$\Rightarrow 2 \cdot \frac{dx}{x} = \left(1 - \frac{3}{y+3}\right) dy$$

Integrating both sides, we get

$$2 \log x = y - 3 \log(y+3) + C$$

$\dots(iii)$

Given that $y(1) = -2$

$\therefore$ From (iii), we have

$$2 \log 1 = -2 - 3 \log(-2+3) + C$$

$$\Rightarrow 2 \cdot 0 = -2 - 3 \cdot 0 + C \Rightarrow C = 2$$

On substituting the value of C in (ii), we get

$$2 \log x = y - 3 \log(y+3) + 2$$

$$\Rightarrow \log x^2 + \log(y+3)^3 = (y+2)$$

$$\Rightarrow x^2 (y+3)^3 = e^{y+2}$$

20. (A) : We have, $y = \log_e(\sin x)$

For $x \in (0, \pi)$, $\sin x \in (0, 1)$

$$\therefore \log_e(\sin x) < 0$$

$\therefore$ Required area,

$$A = - \int_0^{\pi} \log_e \sin x dx = -2 \int_0^{\pi/2} \log_e \sin x dx$$

$$= -2 \frac{\pi}{2} \log_e \frac{1}{2} = \pi \log_e 2 \text{ sq. units}$$

For other sections/subjects please refer to
Physics For You, Chemistry Today and Biology Today





POLICIES/SCHEMES/MISSIONS

Amrit Bharat Station Scheme

The Ministry of Railways has formulated a new scheme for modernization of small stations named “Amrit Bharat Station Scheme”.

- Preparation and implementation of master plans for railway stations to enhance the facilities beyond the minimum essential amenities like creating roof plazas and city centres at the station premises in the long run.
- The scheme shall cater for introduction of new amenities as well as upgradation and replacement of existing amenities.

Mangdechhu Hydroelectric Power Project

- The India-assisted 720 Megawatts Mangdechhu Hydroelectric Power Project has been handed to Druk Green Power Corporation (DGPC) in Bhutan.
- It was jointly inaugurated by Indian Prime Minister Narendra Modi and his Bhutanese Counterpart Lotay Tshering in 2019.
- The Indian Government provided the financial and technical assistance for the construction of this hydroelectric project.
- This is significant, given that the hydropower sector is expected to contribute to Bhutan's economic growth and development based on the philosophy of Gross National Happiness.

Nilgiri Tahr Project

- The state government of Tamil Nadu has launched the Nilgiri Tahr project that aims to protect the state animals.
- The Nilgiri Tahr is an ungulate that is endemic to the southern portion of the Western and Eastern Ghats in Kerala and Tamil Nadu. It is mainly found in the Nilgiri Hills. It is the state animal of Tamil Nadu. It is the only mountain ungulate in southern India among the 12 species found in India.

- The species has been listed as Endangered in the IUCN Red List of Threatened Species. Due to the numerous threats the ungulate faces, it is protected under Schedule I of the Wildlife (Protection) Act of India, 1972.

Project Vaani

- Project Vaani launched to gather speech data from across India for the creation of an Artificial Intelligence (AI) based language model that can understand diverse Indian languages and dialects.
- It is implemented jointly by the Indian Institute of Sciences (IISc), ART PARK (AI and Robotics Technology Park) and Google.

Tamil Nadu Climate Change Mission

- Tamil Nadu recently became the first state in India to launch its own climate change mission. The objective of the mission is to make Tamil Nadu a “climate smart state”.
- The climate change mission document emphasized that climate change is an undisputed reality having severe impact on the natural environment, human lives, economic assets and activities.

Community Innovator Fellowship Program

- The Community Innovator Fellowship (CIF) program is an initiative of the Atal Innovation Mission. It is being implemented in collaboration with the United Nations Development Programme (UNDP) India.
- Its aim is to facilitate knowledge building and provide infrastructure support to aspiring community innovators essential for the entrepreneurship journey.
- This is a one-year-long intensive fellowship program that will provide assistance to innovators involved in solving challenges faced by communities.

New integrated Food Security Scheme

- The Central Government of India recently announced the launch of a new integrated food security scheme, which will provide free food grains to 81.35 crore beneficiaries under the National Food Security Act (NFSA) for the year 2023.
- The Cabinet has decided to implement this central sector scheme in order to fulfil the vision of “One Nation-One Price-One Ration” under Prime Minister Narendra Modi.

India-Australia ECTA

- India-Australia Economic Cooperation and Trade Agreement comes into force from 29 December 2022.
- ECTA is the first trade agreement of India with a developed country after more than a decade. The

agreement encompasses cooperation across the entire gamut of bilateral economic and commercial relations between the two friendly countries.

BIND Scheme

- The Cabinet Committee on Economic Affairs has recently approved a large infrastructure development scheme for Prasar Bharati, the public broadcasting service in India that heads All India Radio and Doordarshan.
- The Broadcasting Infrastructure and Network Development (BIND) scheme, worth over ₹ 2,500 crore, will focus on expanding and upgrading the infrastructure of Prasar Bharati, as well as improving the digital content.

Test Yourself!

1. Which state government has launched the second version of 'Orunodoi', the flagship scheme of the state government?
(a) Tamil Nadu (b) Assam
(c) Kerala (d) Uttar Pradesh
2. Which scheme has now been named as Pradhan Mantri Virasat Ka Samvardhan (PM VIKAS) Scheme?
(a) Pradhan Mantri Jeevan Jyoti Bima Yojana (PMJJBY)
(b) Pradhan Mantri Jan Dhan Yojana (PMJDY)
(c) Pradhan Mantri Kaushal Ko Kaam Karyakram (PMKKK)
(d) Stand Up India Scheme
3. Who has inaugurated the 7th India Water Impact Summit (IWIS 2022) on 15th December, 2022?
(a) Shri Prahlad Singh Patel
(b) Shri Narendra Modi
(c) Shri Gajendra Singh Shekhawat
(d) Shri Parameswaran Iyer
4. The Government of India has approved the Print and Digital Media Association (PADMA) as a self-regulatory body for publishers of news and current affairs across India in December 2022. Who will head the PADMA?
(a) Jitendra Mehta, CEO
(b) Union Minister of I&B, Anurag Thakur
(c) Former HC Judge Mool Chand Garg
(d) Journalist Pankaj Kumar Mishra
5. Recently, government approved a new scheme which aims “to cover all the aspects of Adult Education to align with National Education Policy 2020 and also in line with the Budget Announcement made in 2021-22”. Name the scheme.
(a) New India Adult Literacy Programme
(b) India's Adult Literacy Programme
(c) India Literacy Programme for Adults
(d) New India Literacy Programme
6. Which state has launched “One District One Sport” scheme?
(a) Uttar Pradesh (b) Maharashtra
(c) Kerala (d) Haryana
7. Which programme is launched under the “Deendayal Antyodaya Yojana-National Rural Livelihood Mission (DAY-NRLM)”?
(a) Uchtar Challenge (b) Prachaar Challenge
(c) Ujjawal Challenge (d) Prajjwala Challenge
8. Which state/UT hosted the 'National Organic Food Festival', held recently?
(a) New Delhi (b) Haryana
(c) Punjab (d) Kerala
9. The Union Agriculture Minister launched new features of e-National Agriculture Market (e-NAM) platform amid the lockdown. When was e-NAM launched in India?
(a) 2000 (b) 2005 (c) 2016 (d) 2019
10. Which state launched Convalescent Plasma Therapy for treating COVID-19 patients, named Project PLATINA?
(a) New Delhi (b) Maharashtra
(c) Tamil Nadu (d) Gujarat

Answer Key

1. (b) 2. (c) 3. (c) 4. (c) 5. (d)
6. (a) 7. (d) 8. (a) 9. (c) 10. (b)

Challenging PROBLEMS For JEE

Probability



MORE THAN ONE OPTIONS CORRECT TYPE

- Let us define the events A and B as
 A : An year chosen at random contains 29 February
 B : An year chosen at random has 52 Fridays.
 If $P(E)$, denotes the probability of happening of event E , then
 (a) $P(\bar{B}) = \frac{23}{28}$ (b) $P(B) = \frac{23}{28}$
 (c) $P(A | \bar{B}) = \frac{2}{5}$ (d) $P(A | B) = \frac{5}{23}$
- If the triangles are formed by joining vertices of a octagon, then
 (a) Number of triangle in which exactly one side common with the side of polygon is 32.
 (b) Number of triangle in which atleast one side common with the side of polygon is 48.
 (c) Number of triangle at least one side common with the side of polygon is 50.
 (d) Total number of triangle is 56.
- In a gambling between Mr. A and Mr. B, a machine continues tossing a fair coin until the two consecutive throws either HT or TT are obtained for the first time. If it is HT , then Mr. A wins and if it is TT , then Mr. B wins. Which of the following option(s) is (are) true?
 (a) Probability of winning Mr. A is $3/4$.
 (b) Probability of Mr. B winning is $1/4$.
 (c) Given first toss is head, probability of Mr. A winnings is 1.
 (d) Given first toss is tail, probability of Mr. A winning is $1/2$.
- Ram and Shyam select two numbers from the set 1 to n . If the probability that Shyam selects a number which is less than the number selected by Ram is $\frac{63}{128}$, then
 (a) n is even (b) n is perfect square
 (c) n is a cube (d) None of these
- If M and N are any two events, then the probability that the exactly one of them occurs is
 (a) $P(M) + P(N) - 2P(M \cap N)$
 (b) $P(M) + P(N) - P(M \cap N)$
 (c) $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$
 (d) $P(M \cap N^C) + P(M^C \cap N)$
- A five digit number with distinct digits is formed by using the digit 0, 1, 2, 3, 4, 5. The probability that the number is divisible by 3 is
 (a) $\frac{{}^6P_5}{6^5}$ (b) $\frac{9}{25}$ (c) $\frac{5! + 4 \cdot 4!}{5 \cdot 5!}$ (d) $\frac{{}^6C_5}{6^5}$
- When a coin is flipped ' n ' times and the probability that the first head comes after exactly m ($n > m + 1$) tails is $\frac{1}{2^6}$, then
 (a) $n = 8, m = 5$ (b) $n = 7, m = 5$
 (c) $n = 8, m = 6$ (d) $n = 5, m = 3$

INTEGER ANSWER TYPE

- In a multiple-choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. So he decides to tick answers at random, if the least number of chances, he should be allowed so that the probability of his getting marks on the question exceeds $\frac{1}{8}$ is K , then $K = \underline{\hspace{2cm}}$.
- If $n(X) = (k + 1)$, then the probability of selecting 2 subsets A and B of the set ' X ' such that $B = A^C$ is equal to $\frac{1}{2^{m-1}}$, where $m - k$ is equal to $\underline{\hspace{2cm}}$.

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10. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and seven balls are drawn. The probability that exactly three balls are common to the two drawn is p , then the value of $8p$ is _____.
11. In a group of people, if 4 are selected at a random, the probability that the any two of the four do not have same month of birth is p , then $\frac{96p}{11}$ is equal to _____.
12. Functions are formed from $A = \{1, 2, 3\}$ to set $B = \{1, 2, 3, 4, 5\}$ and one function is selected at random. If p the probability that function satisfying $f(i) \leq f(j)$, whenever $i < j$, then the value of $25p$ is equal to _____.
13. An unbiased coin is tossed 12 times. The probability that at least 7 consecutive heads show up is $\frac{K}{256}$, then $K =$ _____.
14. Two squares are chosen at random on a chess board. If the chance that may have a contact at corner is $\frac{\mu}{144}$, then μ should be equal to _____.
15. A man parks his car among n cars standing in a row, his car not being parked at an end, on his return he finds that exactly m of the n cars are still there, probability that both the cars parked on two sides of his car, have left is $\frac{(n-m)(n-m-1)}{(n-A)(n-B)}$, then $A + B$ is _____.
16. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. If the probability that Mr. A selected the winning horse is $\frac{p}{5}$, then the value of p is _____.
17. In a bag there are 15 balls of either red or green colour. Let G_k be the event that it contains exactly k green balls and its probability is proportional to k^2 . Now a ball is drawn at random. Let $P(A)$ be the probability that the ball drawn is green. If $P(A) = \frac{p}{q}$ is in lowest form, then $q - p$ is _____.
18. The probability that a random chosen 3 digit number has exactly 3 factors is $\frac{p}{900}$ (where $p \in N$), then the value of p is _____.

SOLUTIONS

1. (b, c, d) : $P(A) = \frac{1}{4}, P\left(\frac{B}{A}\right) = \frac{5}{7}, P\left(\frac{\bar{B}}{A}\right) = \frac{6}{7}$

$$P(B) = P(A)P\left(\frac{B}{A}\right) + P(\bar{A})P\left(\frac{B}{\bar{A}}\right) = \frac{1}{4} \times \frac{5}{7} + \frac{3}{4} \times \frac{6}{7} = \frac{23}{28}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4} \times \frac{5}{7}}{\frac{1}{4} \times \frac{5}{7} + \frac{3}{4} \times \frac{6}{7}} = \frac{5}{23}$$

$$P\left(\frac{A}{\bar{B}}\right) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{\frac{1}{4} \times \frac{6}{7}}{1 - \frac{23}{28}} = \frac{2}{5}$$

2. (a, b, d) : Total number of triangles = ${}^8C_3 = 56$
 Number of triangles having exactly one side common with the polygon = $8 \times 4 = 32$
 Number of triangles having exactly two side common with the polygon = 8
 Number of triangles having no side common with the polygon = 16
 Number of triangles in which atmost one side common = exactly one side common + no side common
 = $32 + 16 = 48$

3. (a, b, c, d) : If T comes in first toss, then Mr. B can win in only one case that is TT .
 Probability of Mr. B winning is $1/4$.
 Probability of Mr. A winning is $3/4$.
 Given first toss is head, Mr. A can win in successive tosses are $T, HT, HHT, \dots$

$$\therefore \text{Required probability} = \frac{1/2}{1 - 1/2} = 1$$

Given first toss is tail Mr. A can win in successive tosses are $HT, HHT, HHHT, \dots$

$$\text{Required probability} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1}{2}$$

4. (a, b, c) : Required probability = $\frac{1/2(n^2 - n)}{n^2} = \frac{63}{128}$

$$\Rightarrow 1 - \frac{1}{n} = 1 - \frac{1}{64} \Rightarrow n = 64.$$

5. (a, c, d) : The required probability
 = probability that M occurs and N does not occur or N occurs and M does not occur.
 = $P(M \cap N^C) + P(M^C \cap N)$
 = $P(M) - P(M \cap N) + P(N) - P(M \cap N)$
 = $P(M) + P(N) - 2P(M \cap N)$
 = $1 - P(M^C) + 1 - P(N^C) - 2[1 - P(M \cap N)^C]$
 = $2P(M^C \cup N^C) - P(M^C) - P(N^C)$ [By De-Morgan law]

$$= 2[P(M^C) + P(N^C) - P(M^C \cap N^C)] - P(M^C) - P(N^C) \\ = P(M^C) + P(N^C) - 2P(M^C \cap N^C)$$

6. (b, c) : Let S be the sample space. Then $|S| = 5 \times {}^5P_4$. If 0 is present, then the number of 5 digit number divisible by 3 is $4 \cdot 4! = 96$. If 0 is absent, then the number of 5 digit number divisible by 3 is 120.

$\therefore$ Required probability = $216/600$

7. (a, b) : There are 2^n outcomes in all. The sequence of flips begins with m successive tails followed by a head followed by head or tail.

$\underbrace{TTTTTTTTT}_{m \text{ tails}} \underbrace{-----TH}_{n-(m+1) \text{ tails or heads}} \underbrace{XXX-----X}_{n-(m+1) \text{ tails or heads}}$

$$\therefore \text{Required probability} = \frac{2^{n-(m+1)}}{2^n} = \frac{1}{2^{m+1}} = \frac{1}{2^6}$$

$$\Rightarrow m = 5, n \in \mathbb{N}.$$

8. (4) : The probability that he get marks in first trial = $1/31$
The probability that he get marks in second trial is $\frac{30}{31} \times \frac{1}{30} = \frac{1}{31}$

The probability that he get marks in third trial is $1/31$.
Continuing this process the probability from K trial is

$$\frac{K}{31} > \frac{1}{8} \Rightarrow K > \frac{1}{8} \Rightarrow K = 4.$$

9. (2) : $n(X) = k + 1$

Number of ways to construct $A = 2^{k+1}$

Number of ways to construct $B = 2^{k+1}$

$\therefore$ Total ways to construct A and $B = 2^{k+1} \times 2^{k+1}$

Total favorable ways to construct $A = 2^{k+1}$

Favorable ways to construct B such that $B = A^C$

corresponding to every A is = 1

$\therefore$ Favourable ways = $2^{k+1} \times 1$

$$\therefore \text{Required Probability} = \frac{2^{k+1}}{(2^{k+1})^2} = \frac{1}{2^{k+1}}$$

$$\Rightarrow m - 1 = k + 1 \Rightarrow m - k = 2$$

10. (7) : The number of ways of drawing 7 balls = ${}^{10}C_7$
For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the first draw, i.e., 2 other balls can be drawn in ${}^3C_2$ ways thus, for each set of 7 balls of the second draw, there are ${}^7C_3 \times {}^3C_2$ ways of making the first draw so that there are 3 balls common. Hence, the probability of having three balls in common

$$= \frac{{}^7C_3 \times {}^3C_2}{{}^{10}C_7} = \frac{7}{8}.$$

$$\text{11. (5) : Required probability} = \frac{{}^{12}C_4 \cdot 4!}{12^4} = \frac{55}{96}$$

12. (7) : Total number of function = $5^3 = 125$

$$\text{Number of function satisfying } f(i) \leq f(j) \text{ if } i < j \\ = {}^5C_3 + {}^5C_2(1 + 1) + {}^5C_1 = 35$$

$$\text{Required probability} = \frac{35}{125} = \frac{7}{25}$$

13. (7) : The sequence of consecutive heads may starts with 1st toss or 2nd toss or 3rd toss or at 6th toss. In any case, if it starts with r^{th} throw, the first $(r - 2)$ throws may be head or tail but $(r - 1)^{\text{th}}$ throw must be tail, after which again a head or tail can show up.

$\underbrace{HHH---H}_{7 \text{ times}} \underbrace{XXXXX}_{(r-2)} \text{ or } \underbrace{XXX---X}_{(r-2)} \underbrace{T}_{(r-1)^{\text{th}}} \underbrace{HHH---H}_{HXX-----}$

$\therefore$ Required probability

$$= \frac{1}{2^7} + \frac{1}{2} \cdot \frac{1}{2^7} + \frac{1}{2^7} + \dots + \frac{1}{2} \cdot \frac{1}{2^7} = \frac{1}{2^7} \left[1 + \frac{5}{2} \right] = \frac{7}{2^8}$$

14. (7) : Total cases of choosing two squares on a chess board = 64×63

Favorable cases = $4 \times 1_{(\text{corners})} + 24 \times 2 + 36 \times 4 = 4 \times 49$

$$\text{Required probability} = \frac{4 \times 49}{64 \times 63} = \frac{7}{144} \Rightarrow \mu = 7$$

15. (3) : Number of ways in which remaining $m - 1$ cars can take their places (excluding the car of man) = ${}^{n-1}C_{m-1}$

Number of ways in which remaining $(m - 1)$ cars can take places keeping the two places on two sides of his car vacant = ${}^{n-3}C_{m-1}$

$$\therefore \text{Required Probability} = \frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}} = \frac{(n-m)(n-m-1)}{(n-1)(n-2)}$$

$$\Rightarrow A = 1, B = 2 \therefore A + B = 3$$

16. (2) : Out of 5 horses only one is the winning horse. The probability that Mr. A selected both the loss horses = $\frac{4}{5} \times \frac{3}{4}$

$\therefore$ The probability that Mr. A selected the winning horse = $1 - \frac{4}{5} \cdot \frac{3}{4} = \frac{2}{5}$

$$\text{17. (7) : } P(G_k) \propto k^2 \Rightarrow P(G_k) = \lambda k^2; \sum_{k=0}^n P(G_k) = 1$$

(as these are mutually exclusive and exhaustive events)

$$\Rightarrow \lambda \sum_{k=0}^n k^2 = 1 \Rightarrow \lambda = \frac{6}{n(n+1)(2n+1)}$$

$$P(A) = \sum_{k=0}^n P(G_k) P\left(\frac{A}{G_k}\right) \\ = \sum_{k=0}^n \lambda k^2 \cdot \frac{k}{n} = \frac{\lambda}{n} \cdot \frac{n^2(n+1)^2}{4} = \frac{3(n+1)}{2(2n+1)}$$

$$\text{For } n = 15, P(A) = \frac{24}{31}$$

18. (7) : A number has exactly 3 factors if the number is squares of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3-digit numbers

$\therefore$ Required probability = $7/900$.



UCD

Unique Career in Demand

Explore the available Unique Career Options!



B.Sc. (Hons.) in Statistics

B.Sc. (Hons.) in Statistics is a 3-year Undergraduate course after 12th divided into 6 semesters, allowing the students to understand, examine and use the knowledge of statistics and arithmetic to have a successful career. B.Sc. Statistics emphasises the areas of Probability and Statistical Methods, Survey Sampling, Numerical Analysis, etc. The course includes lectures, flying and simulation training, assignments, etc aimed at offering classroom and practical training as well.

Job Perspectives

B.Sc. (Hons.) Statistics jobs are present as an assistant professor, subject matter expert, marketing research analyst, Statistics manager and many more. There is a wide range of work opportunities for aspirants after this course in both public and private sector. Some of which are as follows:

- Actuarial Manager
- Research Assistant Manager
- Marketing Assistant Manager
- Credit Risk Strategist
- Statistics Trainer

Eligibility

The students planning to pursue a B.Sc. (Hons.) in Statistics must have completed 10th with subjects Physics, Chemistry and Mathematics with at least 50% marks from a recognized school in India.

Admission Process

- Merit Based: Some colleges offer admission on the basis of merit of their overall score in Intermediate Exams.
- Entrance Based: Some colleges offer admission to B.Sc. (Hons.) Statistics on the basis of marks obtained in the Entrance Exams. After qualifying the entrance exams, the shortlisted students may also require to appear for a Group Discussion and Personal Interview round.

Entrance Exams

NPAT, CUET, CUCET, SET

Top Colleges

1. Hindu College, New Delhi.
2. VIT Vellore,
3. Delhi University
4. Aligarh Muslim University
5. Cochin University of Science and technology
6. St Xavier's College, Ahmedabad.
7. Ramjas College, New Delhi

College Info

Hindu College is a constituent college of the University of Delhi in New Delhi, India. It offers both undergraduate and postgraduate programmes in sciences, humanities, social sciences and commerce. In 2022, it is ranked 2nd nationally by National Institute Ranking Framework (NIRF) under Ministry of Human Resource Development (Government of India). At the undergraduate level in Delhi University, Statistics at Hindu College was the first in its league. It premiered in the year 1975, post which others followed this suit in the subsequent years. Candidates must appear in CUET exam for the admission to this course. B.Sc. (Hons.) in Hindu college. The department has a permanent faculty of seven committed teachers who specialize in various academic fields of statistics like Inference, Stochastic Modeling, Prediction Theory, Biostatistics, etc.

UNSCRAMBLE ME

Unscramble the words given in column I and match them with their explanations in column II.

Column I

1. PANONISXE
2. AIONTEDVI
3. ACTNOIFR
4. PIPARAPEDLLELE
5. LOPES
6. CEORSSDEERP
7. EZITMUPRA
8. GELAABR
9. TISCSTATIS
10. PHSEER

Column II

- (a) The ratio of change in y -coordinate and change in x -coordinate.
- (b) The sets of all points in three dimensional space lying at the same distance from a given point.
- (c) The branch of mathematics for collecting, analysing and interpreting data.
- (d) The branch of mathematics that represents problems in the form of mathematical expressions.
- (e) A four sided shape which has one pair of parallel sides.
- (f) The three dimensional figure made up of six parallelograms.
- (g) A numerical representation for part of a whole.
- (h) Anything that varies from the accepted or standard norm.
- (i) A number or other mathematical expression written in an expanded form.
- (j) A number that comes immediately before the original number.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.



Unlock Your Knowledge!



1. Write the coefficient of y in the expansion of $\left(y^2 + \frac{c}{y}\right)^5$.

2. Find the value of $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{9}$.

3. Find the perpendicular distance from the point $(3, -4)$ to the line $3x - 4y + 10 = 0$.

4. Find the value of $\lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{x}$.

5. Four dice are rolled. Find the number of possible outcomes in which at least one dice shows 2.

6. Three numbers form an increasing G.P. If the middle term is doubled, then the new numbers are in A.P. Find the common ratio of G.P.

7. If the sum of 10 items is 12 and the sum of their squares is 18, then find the standard deviation.

8. If the angles of a triangle are in the ratio $1 : 4 : 5$, then find the ratio of the greater side to the smallest side.

9. Find the value of $(A')'$.

10. What is the locus of a point, whose abscissa and ordinate are always equal?

11. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then find the value of $f \circ g(x)$.

12. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then find the value of $x^4 + y^4 + z^4 + 4x^2y^2z^2$.

13. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then find the value of x .

14. For the non-singular matrix A , if $A^2 = I$, then find A^{-1} .

15. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then find $\frac{dy}{dx}$.

16. If $\sin^p \theta \cos^q \theta$ attain maximum, then find the value of θ .

17. Evaluate: $\int_1^2 x^2 dx$

18. Solve: $\frac{xdy}{dx} = y(\ln y - \ln x + 1)$

19. Find a vector of magnitude 5 units and perpendicular to $\hat{i} - 2\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$.

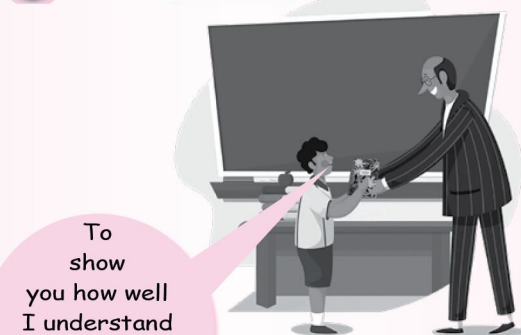
20. Find the angle between the lines

$$\frac{x-5}{1} = \frac{y-7}{2} = \frac{z-4}{3} \text{ and } \frac{x}{2} = \frac{y+3}{2} = \frac{z-4}{-2}.$$

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.



COMIC CAPSULE



To show you how well I understand fractions, I did only half of my homework.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. Find the general solution of the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$. (Anand, W.B.)

Ans. We have, $(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x \cos^2 2x = \sin 2x \cos 2x$

$$\Rightarrow 2\sin^2 2x \cos^2 2x + \sin 2x \cos 2x - 1 = 0$$

Put $\sin 2x \cos 2x = t$.

Therefore, we have, $2t^2 + t - 1 = 0$

$$\Rightarrow (2t - 1)(t + 1) = 0 \Rightarrow t = \frac{1}{2}, -1$$

Case I : When $t = -1$, we have $\sin 2x \cos 2x = -1$

$\Rightarrow \sin 4x = -2$, which is impossible.

$\therefore t = -1$ is to be rejected.

Case II : When $t = 1/2$, we have $\sin 4x = 1$

$$\Rightarrow 4x = 2n\pi + (\pi/2)$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8} = (4n + 1)\frac{\pi}{8}$$

2. If p^{th} , q^{th} and r^{th} terms of an H.P., be respectively x , y and z , then prove that $(p - q)xy + (q - r)yz + (r - p)zx = 0$. (Ankit Jain, M.P.)

Ans. Since, x , y , z are respectively the p^{th} , q^{th} and r^{th} terms of an H.P., hence, $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ will respectively be the

p^{th} , q^{th} and r^{th} terms of an A.P.

If a and d be the first term and common difference of the corresponding A.P., then we have

$$\frac{1}{x} = a + (p - 1)d \quad \dots(i)$$

$$\frac{1}{y} = a + (q - 1)d \quad \dots(ii)$$

$$\text{and } \frac{1}{z} = a + (r - 1)d \quad \dots(iii)$$

Subtracting (ii) from (i), we have

$$\frac{1}{x} - \frac{1}{y} = (p - q)d$$

$$\text{i.e., } xy(p - q) = \frac{y - x}{d} \quad \dots(iv)$$

Similarly, subtracting (iii) from (ii), we have,

$$yz(q - r) = \frac{z - y}{d} \quad \dots(v)$$

Subtracting (i) from (iii), we have,

$$zx(r - p) = \frac{x - z}{d} \quad \dots(vi)$$

Adding (iv), (v) and (vi), we get the desired result.

3. Find the projection of the line $\frac{x-1}{0} = \frac{y-3}{1} = \frac{z-4}{7}$ in the plane $2x - y + z + 3 = 0$. (Suprabha, Chennai)

Ans. Given line is $\frac{x-1}{0} = \frac{y-3}{1} = \frac{z-4}{7} = \lambda$ (say).

Coordinate of general point on the line is $(1, \lambda + 3, 7\lambda + 4)$.

Its foot on the plane $2x - y + z + 3 = 0$ is

$$\frac{x-1}{2} = \frac{y-\lambda-3}{-1} = \frac{z-7\lambda-4}{1} = \frac{-(2-\lambda-3+7\lambda+4+3)}{4+1+1} = -(\lambda+1)$$

$$\Rightarrow x - 1 = -2\lambda - 2, y - 3 = 2\lambda, z = 6\lambda + 3$$

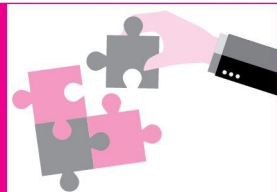
$$\Rightarrow x + 1 = -2\lambda, y - 3 = 2\lambda, z - 3 = 6\lambda$$

$$\Rightarrow \frac{(x+1)}{-1} = \frac{(y-3)}{1} = \frac{(z-3)}{3}, \text{ which is the projection of the line on given plane.}$$



PUZZLE CORNER

ANSWER - JANUARY 2023



14+	5	6	4÷	1	5+	3	2	20×	4
36×	2	3	4	1	6×	1	6	5	
3	13+	5	2	6	12×	4	1		
6	8×	4	10×	5	2	12+	1	3	
1	2	12+	3	4	5	6			
11+	4	1	6	5	5+	3	2		

Competency Based Questions

for CBSE BOARD EXAM 2022-23



As per the circular issued by CBSE for Assessment and Evaluation practices for the session 2022-23, the Internal/ year end Board Examination questions paper will be having minimum 40% Competency Based Questions. These can be in the form of Multiple Choice Questions, Assertion & Reason based, Integrated or any other types of questions. In this article, we have specially created Some Competency Based Questions for student for maximum practice.

MCQs

Multiple Choice Questions

1. Let R be an equivalence relation on a finite set A having n elements. Then, the number of ordered pairs in R is
 - (a) Less than n
 - (b) Greater than or equal to n
 - (c) Less than or equal to n
 - (d) None of these
2. The mapping $f : N \rightarrow N$ given by $f(n) = 1 + n^2$, $n \in N$ where N is the set of natural numbers, is
 - (a) one-one and onto
 - (b) onto but not one-one
 - (c) one-one but not onto
 - (d) neither one-one nor onto
3. The domain of the function $\cos^{-1}(2x - 1)$ is
 - (a) $[0, 1]$
 - (b) $[-1, 1]$
 - (c) $(-1, 1)$
 - (d) $[0, \pi]$
4. $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is equal to
 - (a) $\frac{\pi}{6}$
 - (b) $\frac{\pi}{3}$
 - (c) $\frac{4\pi}{3}$
 - (d) $\frac{3\pi}{4}$
5. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the values of a and b respectively.
 - (a) 2, 4
 - (b) 4, 2
 - (c) Both (a) and (b)
 - (d) None of these
6. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$, then A is a
 - (a) symmetric matrix
 - (b) skew-symmetric matrix
 - (c) diagonal matrix
 - (d) None of these
7. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
 - (a) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$
 - (b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
 - (c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
 - (d) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

8. Find x , if $\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is singular.

- (a) 1 (b) 2
(c) 3 (d) 4

9. If $f(x) = -\sqrt{25-x^2}$, then $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$ is equal to

- (a) $\frac{1}{24}$ (b) $\frac{1}{5}$
(c) $-\sqrt{24}$ (d) $\frac{1}{\sqrt{24}}$

10. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{dy}{dx}$ equals

- (a) $\frac{hx+by}{ax+hy}$ (b) $\frac{ax+hy}{hx+by}$
(c) $\frac{ax+hx}{hy+by}$ (d) $\frac{-(ax+hy)}{hx+by}$

11. The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on

- (a) $(-1, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty)$ (d) None of these

12. If x is real, then the minimum value of $x^2 - 8x + 17$ is

- (a) -1 (b) 0
(c) 1 (d) 2

13. Evaluate : $\int [\sec^2(7-4x)] dx$

- (a) $-\frac{1}{4} \tan(7-4x) + C$
(b) $\frac{1}{4} \tan(7-4x) + C$
(c) $\frac{1}{4} \tan(7+4x) + C$
(d) $-\frac{1}{4} \tan(7x-4) + C$

14. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

- (a) 2 sq. units (b) 4 sq. units
(c) 3 sq. units (d) 1 sq. unit

15. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{2/3} + 4 - 3\frac{dy}{dx} = 0$$
 is

- (a) 2 (b) 1
(c) 3 (d) None of these

16. Integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

- (a) x (b) $\log x$
(c) $\frac{1}{x}$ (d) $-x$

17. If O is origin and C is the mid point of $A(2, -1)$ and $B(-4, 3)$, then the value of $\overrightarrow{OC}$ is

- (a) $\hat{i} + \hat{j}$ (b) $\hat{i} - \hat{j}$
(c) $-\hat{i} + \hat{j}$ (d) $-\hat{i} - \hat{j}$

18. If the directions cosines of a line are k, k, k , then

- (a) $k > 0$ (b) $0 < k < 1$
(c) $k = 1$ (d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

19. Corner points of the feasible region determined by the system of linear constraints are $(0, 3)$, $(1, 1)$ and $(3, 0)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$ is

- (a) $p = 2q$ (b) $p = \frac{q}{2}$
(c) $p = 3q$ (d) $p = q$

20. If A and B are 2 events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\bar{A} | \bar{B}) =$

- (a) $1 - P(A|B)$ (b) $1 - P(A | \bar{B})$
(c) $\frac{1 - P(A \cup B)}{P(B)}$ (d) $\frac{P(\bar{A})}{P(B)}$

A & R

Assertion & Reason Based Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

1. **Assertion (A)** : The relation R in a set $A = \{1, 2, 3, 4\}$ defined by $R = \{(x, y) : 3x - y = 0\}$, $x \in A$, have the Domain = $\{1, 2, 3, 4\}$ and Range = $\{3, 6, 9, 12\}$.

Reason (R) : Domain & Range of the relation (R) is respectively the set of all second & first entries of the distinct ordered pair of the relation.

2. **Assertion (A)** : Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$, where $f: A \rightarrow B$ given by $f(x) = x^2$, then f is a many-one function.

Reason (R) : If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in \text{domain}$, then f is one-one or else many-one.

3. **Assertion (A)** : $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

Reason (R) : $A = [a_{ij}]_{n \times n}$ is a square matrix such that $a_{ij} = 0$, $\forall i \neq j$, then A is called diagonal matrix.

4. **Assertion (A)** : If $2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$, then x has 2 solutions.

Reason (R) : $\sin^{-1}(\sin x) = x$, if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

5. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$.

Assertion (A) : $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Reason (R) : $P(A \cup B) = P(A) + P(B)$.

6. **Assertion (A)** : If $y = \log_{10}x + \log_e x$, then

$$\frac{dy}{dx} = \frac{\log_{10} e}{x} + \frac{1}{x}$$

Reason (R) : $\frac{d}{dx}(\log_{10} x) = \frac{\log x}{\log 10}$

$$\text{and } \frac{d}{dx}(\log_e x) = \frac{\log x}{\log e}$$

7. **Assertion (A)** : If the function $f(x) = \frac{ae^x + be^{-x}}{ce^x + de^{-x}}$

is increasing function of x , then $bc > ad$.

Reason (R) : A function $f(x)$ is increasing if $f'(x) > 0$ for all x .

8. **Assertion (A)** : The vector perpendicular to both

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \text{ and}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3} \text{ is } -\hat{i} - 7\hat{j} + 5\hat{k}.$$

Reason (R) : The vector perpendicular to the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is $\vec{b}_1 \times \vec{b}_2$.

9. **Assertion (A)** : $\int_{-\pi/2}^{\pi/2} |\sin x| dx = 2$

$$\text{Reason (R)} : \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx,$$

where $c \in (a, b)$.

10. **Assertion (A)** : The area bounded by the curve $y = 2\cos x$ and the x -axis from $x = 0$ to $x = 2\pi$ is 8 sq. units.

Reason (R) : If $f(x)$ is a continuous function defined on $[a, b]$ then the area bounded by the curve $y = f(x)$, the x -axis and the ordinate $x = a$ and $x = b$ is $\left| \int_a^b f(x) dx \right|$.

11. **Assertion (A) :** 'x' is not an integrating factor for the differential equation $x \frac{dy}{dx} + 2y = e^x$.

Reason (R) : For the differential equation of type $\frac{dy}{dx} + py = Q$, I.F is $e^{\int p dx}$.

12. Let $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$.

Assertion (A) : Vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Reason (R) : $\vec{a} \cdot \vec{b} = 0$

13. **Assertion (A) :** The length of projection of the vector $3\hat{i} - \hat{j} - 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} - 3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

Reason (R) : The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a} \cdot \vec{b})}{|\vec{b}|}$.

14. Let E_1 and E_2 be any two events associated with an experiment, then

Assertion (A) : $P(E_1) + P(E_2) \leq 1$.

Reason (R) : $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$.

15. **Assertion (A) :** Let E and F be events associated with the sample space S of an experiment. Then, we have $P(S|F) = P(F|F) = 1$.

Reason (R) : If A and B are any two events associated with the sample space S and F is an event associated with S such that $P(F) \neq 0$, then $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$



Case Based Questions

1. To improve the air quality of Delhi, students of a particular school wants to plants some trees in different societies parks.



The local nursery charges the cost of plants by the following formula, $c(x) = x^3 - 45x^2 + 600x$, where x is the number of trees and $c(x)$ is the cost of plants (in ₹). School principal, while giving them instructions, imposed a restriction that they can plant 10 to 20 trees in one particular park for a fair distribution.

Based the above information, answer the following questions.

- Find the derivative of $c(x)$ w.r.t 'x'.
- What are the possible number of trees if $c'(x) = 0$?
- For how many trees should they place the order for one park so that their expenses are least?

OR

Find the least amount (in ₹), that the children has to spend?



CBSE CHAPTERWISE TOPICWISE 10 SOLVED PAPERS (2022-2013)



CBSE CHAMPION Chapterwise - Topicwise Solved Papers Series contains topicwise questions and solutions asked over last decade in CBSE-Board examination.

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HIGHLIGHTS OF 2022-23 EDITION

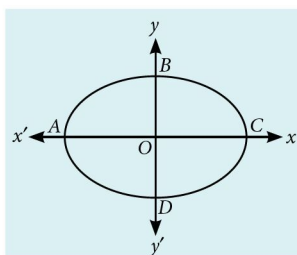
- Based on Reduced Syllabus
- Topicwise Graphical Analysis
- Quick Recap with Brain Map
- Previous 10 years (2022- 2013) questions of CBSE
- CBSE Cognitive Level Tagging
- Key Points, Answer Tips, Concept Applied, Shortcuts, Alternating Methods, Commonly Made Mistakes provided
- CBSE Topper's Answers
- Past 3 years CBSE Sample Question Papers
- Chapterwise Self-Assessment
- 5 Practice Papers
- CBSE Sample Paper (Issued on 16th September 2022)



2. Pinku visited the society park in the evening with his father. Father bought an ice cream for Pinku, which was very delicious.



Assume that shape of ice-cream is described by the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$, when it is kept on the xy plane. Using the above information answer the following questions.



- Find the area of part $AOCBA$ using integration.
- Find the area of whole shape of ice-cream.

3. For an audition of a reality dance competition, interested candidates were asked to apply under one of the two dance forms : Modern or contemporary, under one of the two age categories below 15 or 15 and above.



Modern Dance



Contemporary Dance

Total 2000 applications were received in which

- 960 applications were for modern dance.
- 192 applications of modern dance were for the below 15 category
- 104 applications of contemporary dance were for the 15 and above category.

Based on the above information answer the following questions.

- (i) What is the probability that an application selected at random is for 15 and above category provided it is under the contemporary dance?
- (ii) An application selected at random is found to be under the below 15 category. Find the probability that it is under the modern dance form.

4.

Ram singh observed the problem of stray animals, wandering into fields attacking children or older people and sometimes dying on the street side. He decided to donate a rectangular plot of land for these stray animals to village Pachayat to make animal shelter. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breath is increased by 50 m, then its area will remain same. But if its length is decreased by 10 m and breath is decreased by 20 m, then is area will decrease by 5300 m^2 .



Rectangular Plot

Based on the above information, answer the following questions.

- (i) Write the system of equations represented in given situation in terms of x and y .
- (ii) Write the matrix equation represents the information given above.
- (iii) Find the length and breadth of the rectangular field.

OR

How much is the area of the rectangular field?

5.

Mrs. Navpreet decided to try yet another engaging style of learning and selected a group of students who would replace her to conduct the class on a particular topic.



For this purpose she write the following relations defined on set $A = \{1, 2, 3\}$

$$R_1 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_3 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 3), (2, 1), (3, 2)\}$$

The group of students were asked to use their group member to explain the various properties of relations. All who answer the following questions correctly is selected for a homework free week.

- Which relation is reflexive and transitive but not symmetric?
- Which relation is reflexive and symmetric but not transitive?

6. Amit's father started a new agency of cars and take a dealership of particular company. As he has some money to invest and has a space for storage of few items.

Let x denote the number of diesel cars and y denote the number of electric cars purchased by Amit's father in a particular month.



Constraints related to investment is given by $3x + 2y \leq 48$. and objective function is given by $z = 22x + 18y$.

Some other constraints consists are $x + y \leq 20$; $x, y \geq 0$

Based on the above information, answer the following questions.

- Find the number of corner points of the feasible region.
- Find the sum of values of z at all the corner points.
- To get the maximum profit, how many electric cars should be purchased?

OR

To get the maximum profit how many diesel cars should be purchased?

7. Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.



A



B

Based on the above information, answer the following questions.

- Find the cartesian equation of the line along which motorcycle A is running.
- Find the shortest distance between the gives two lines.

SOLUTIONS

MULTIPLE CHOICE QUESTIONS (MCQS)

1. (b): As R is an equivalence relation on set A .

Hence, R has at least n ordered pairs.

2. (c): Since, $f(n) = 1 + n^2$

For one-one, $1 + n_1^2 = 1 + n_2^2$, $n_1, n_2 \in N$

$$\Rightarrow n_1^2 - n_2^2 = 0 \Rightarrow n_1 = n_2 \quad (\because n_1 + n_2 \neq 0)$$

$\therefore f(n)$ is one-one.

Clearly, $f(n)$ is not onto.

3. (a): Let $y = \cos^{-1}(2x - 1) \Rightarrow \cos y = 2x - 1$

Now, $-1 \leq 2x - 1 \leq 1$

$$\Rightarrow 0 \leq 2x \leq 2 \Rightarrow 0 \leq x \leq 1$$

Hence, the domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$.

4. (c): $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$= \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} + 4 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}$$

5. (c): Since, $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$$\Rightarrow a + b = 6 \text{ and } ab = 8$$

$$\Rightarrow a + \frac{8}{a} = 6 \quad (\because ab = 8 \Rightarrow b = 8/a)$$

$$\Rightarrow a^2 - 6a + 8 = 0 \Rightarrow (a - 2)(a - 4) = 0 \Rightarrow a = 2, 4$$

Hence, $a = 2, b = 4$ or $a = 4, b = 2$

6. (b): $A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -A$

Since $A^T = -A$, therefore, A is a skew-symmetric matrix.

7. (b): We have, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\therefore \text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

8. (d): For singular matrix, $|A| = 0$

$$\therefore \begin{vmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(-1 - 1) - 2(-1 - 2) + x(1 - 2) = 0$$

$$\Rightarrow -2 + 6 - x = 0 \Rightarrow x = 4$$

9. (d): $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = f'(1)$

$$\text{Now, } f'(x) = -\frac{1}{2} \cdot \frac{-2x}{\sqrt{25 - x^2}}$$

$$\therefore f'(1) = -\frac{1}{2} \cdot \frac{-2 \times 1}{\sqrt{25 - 1^2}} = \frac{1}{\sqrt{24}}$$

10. (d): Given, $ax^2 + 2hxy + by^2 = 1$

Differentiating w.r.t. x , we get

$$2ax + 2h\left(x \frac{dy}{dx} + y\right) + 2by \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-(ax + hy)}{hx + by}$$

11. (a): Given, $f(x) = \log(1 + x) - \frac{2x}{2 + x}$

$$\Rightarrow f'(x) = \frac{1}{1 + x} - \frac{(2 + x)2 - 2x}{(2 + x)^2} \Rightarrow f'(x) = \frac{x^2}{(x + 1)(x + 2)^2}$$

Clearly, $f'(x) > 0$ for all $x > -1$, hence $f(x)$ is increasing on $(-1, \infty)$.

12. (c): Let $f(x) = x^2 - 8x + 17$

$$\therefore f'(x) = 2x - 8$$

$$\text{Now, } f'(x) = 0 \Rightarrow x = 4$$

$$\text{Also, } f''(x) = 2 > 0, \forall x$$

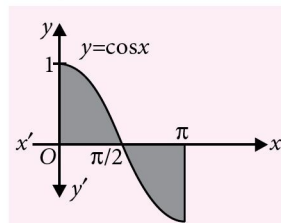
Therefore, $x = 4$ is the point of local minima.

Hence, minimum value of $f(x)$ at $x = 4$ is

$$f(4) = (4 \times 4) - (8 \times 4) + 17 = 1$$

13. (a): $\int \sec^2(7 - 4x) dx = -\frac{1}{4} \tan(7 - 4x) + C$

14. (a): We have, $y = \cos x$



$\therefore$ Required area = area of shaded region

$$= 2 \int_0^{\pi/2} \cos x \, dx = 2 [\sin x]_0^{\pi/2} = 2 \text{ sq. units}$$

$$15. (a): \left(\frac{d^2y}{dx^2}\right)^{2/3} = 3\frac{dy}{dx} - 4 \Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left(3\frac{dy}{dx} - 4\right)^3$$

$\therefore$ Degree is 2.

$$16. (c): \frac{xdy}{dx} - y = x^4 - 3x$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{x^4 - 3x}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x} \cdot y = x^3 - 3$$

$$\therefore \text{I.F.} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

17. (c): Since, C is the mid point of A(2, -1) and B(-4, 3).

$$\therefore \text{Coordinates of C is } \left(\frac{2-4}{2}, \frac{-1+3}{2}\right) = (-1, 1)$$

$$\therefore \overrightarrow{OC} = -\hat{i} + \hat{j}$$

18. (d): If (k, k, k) are direction cosines of a line, then $k^2 + k^2 + k^2 = 1 \Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \Rightarrow k = \pm \frac{1}{\sqrt{3}}$

19. (b): We must have value of Z at (3, 0) = value of Z at (1, 1)

$$\Rightarrow 3p + 0 \cdot q = 1 \cdot p + 1 \cdot q \Rightarrow 3p = p + q \Rightarrow p = \frac{1}{2}q$$

$$20. (b): P(A | \bar{B}) + P(\bar{A} | \bar{B}) = 1 \Rightarrow P(\bar{A} | \bar{B}) = 1 - P(A | \bar{B})$$

ASSERTION / REASON BASED QUESTIONS

1. (c): $R = \{(x, y) : y = 3x, x \in A\}$

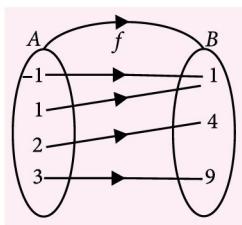
$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

Domain of the relation = $\{1, 2, 3, 4\}$

Range of the relation = $\{3, 6, 9, 12\}$.

$\therefore$ A is true but R is false.

2. (a): Here $f(-1) = 1, f(1) = 1, f(2) = 4, f(3) = 9$



Two elements 1 and -1 have the same image $1 \in B$.

So, f is a many-one function.

Both A and R are true and R is the correct explanation of A.

3. (a): If $A = [a_{ij}]_{n \times n}$ is a square matrix such that $a_{ij} = 0$ for $i \neq j$; then A is called diagonal matrix. Thus,

the given statement is true and $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

$\therefore$ Both A and R are true and R is the correct explanation of A.

$$4. (d): 2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$$

$$\Rightarrow \sin^{-1}x = \frac{5 \pm \sqrt{25-16}}{4}$$

$$\Rightarrow \sin^{-1}x = \frac{1}{2}, \sin^{-1}x = 2$$

$$\Rightarrow x = \sin\left(\frac{1}{2}\right) \text{ is only solution}$$

$$[\because \frac{-\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}, \sin^{-1}x = 2 \text{ is not possible}]$$

$\therefore$ A is false but R is true.

5. (c): Since, $P(A \cap B) = P(A)P(B)$, therefore, A and B are independent events.

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

Similarly, $P(B|A) = P(B)$.

Thus, A is true.

However, R is not true for independent events.

6. (c): We have, $y = \log_{10}x + \log_e x$

$$\therefore \frac{dy}{dx} = \frac{1}{x} \log_{10} e + \frac{1}{x}$$

$\therefore$ A is true.

However, R is not true.

$$7. (d): f'(x) = \frac{2(ad-bc)}{(ce^x + de^{-x})^2}$$

and $f(x)$ is an increasing function.

$$\therefore f'(x) > 0 \Rightarrow \frac{2(ad-bc)}{(ce^x + de^{-x})^2} > 0$$

$$\Rightarrow 2(ad-bc) > 0 \Rightarrow ad > bc \Rightarrow bc < ad$$

$\therefore$ A is false but R is true.

8. (a): The direction ratios of L_1 and L_2 are (3, 1, 2) and (1, 2, 3) respectively and the vector perpendicular

$$\text{to both is given by } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$\therefore$ Both A and R are true and R is the correct explanation of A.

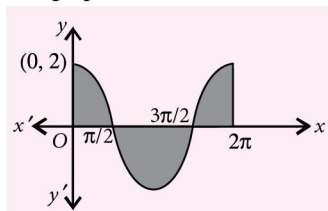
9. (b): $\because |\sin x|$ is an even function

$$\begin{aligned}\therefore \int_{-\pi/2}^{\pi/2} |\sin x| dx &= 2 \int_0^{\pi/2} \sin x dx \\ &= -2[\cos x]_0^{\pi/2} = -2(0-1) = 2\end{aligned}$$

Hence, both A and R are true but R is not the correct explanation of A.

10. (b): We have, $y = 2\cos x$

Let us draw the graph of $2\cos x$ between 0 to 2π .



$\therefore$ Required area = area of shaded region

$$\begin{aligned}&= \int_0^{\pi/2} 2\cos x dx + \left| \int_{\pi/2}^{3\pi/2} 2\cos x dx \right| + \int_{3\pi/2}^{2\pi} 2\cos x dx \\ &= 2\left[\sin x\right]_0^{\pi/2} + \left| 2\left[\sin x\right]_{\pi/2}^{3\pi/2} \right| + 2\left[\sin x\right]_{3\pi/2}^{2\pi} \\ &= 2\left[\sin \frac{\pi}{2} - 0\right] + \left| 2\left[\sin \frac{3\pi}{2} - \sin \frac{\pi}{2}\right] \right| + 2\left[\sin 2\pi - \sin \frac{3\pi}{2}\right] \\ &= 2 + 2 \times 2 + 2 = 2 + 4 + 2 = 8 \text{ sq. units}\end{aligned}$$

Hence, both A and R are true but R is not the correct explanation of A.

11. (a): $\frac{dy}{dx} + \frac{2}{x}y = \frac{e^x}{x}$

I.F. = $e^{\int \frac{2}{x} dx} = e^{\log x^2} = x^2$

$\therefore$ Assertion is true.

Hence, both A and R are true and R is the correct explanation of A.

12. (a): $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} + \hat{j} - 3\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k}) \\ &= 1 \cdot 2 + 1 \cdot 1 + (-3) \cdot 1 = 2 + 1 - 3 = 0\end{aligned}$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Hence, $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Hence, both A and R are true and R is the correct explanation of A.

13. (a): Required length = $\left| \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1^2 + 2^2 + (-3)^2}} \right|$

$$= \left| \frac{3 - 2 + 6}{\sqrt{1 + 4 + 9}} \right| = \frac{7}{\sqrt{14}}$$

Also, vector projection of $\vec{a}$ on $\vec{b} = (\vec{a} \cdot \hat{b}) = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right)$

Hence, both A and R are true and R is the correct explanation of A.

14. (d): Reason is a standard result. It is addition theorem of probabilities. However, the assertion is not true as we have $P(E_1) + P(E_2) > 1$

For example, when a dice is rolled once and

E_1 : 'a number < 5 ' shows up,

E_2 : 'a number > 1 ' shows up

then $P(E_1) = \frac{4}{6} = \frac{2}{3}$ and also $P(E_2) = \frac{5}{6}$.

Here, $P(E_1) + P(E_2) > 1$.

Hence, A is false but R is true.

15. (b): We know that,

$$P(S|F) = \frac{P(S \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Also, $P(F|F) = \frac{P(F \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

Thus, $P(S|F) = P(F|F) = 1$

We have,

$$\begin{aligned}P((A \cup B)|F) &= \frac{P[(A \cup B) \cap F]}{P(F)} \\ &= \frac{P[(A \cap F) \cup (B \cap F)]}{P(F)}\end{aligned}$$

(by distributive law of intersection over union)

$$\begin{aligned}&= \frac{P(A \cap F) + P(B \cap F) - P(A \cap B \cap F)}{P(F)} \\ &= \frac{P(A \cap F)}{P(F)} + \frac{P(B \cap F)}{P(F)} - \frac{P((A \cap B) \cap F)}{P(F)}\end{aligned}$$

$$= P(A|F) + P(B|F) - P((A \cap B)|F)$$

Hence, both A and R are true but R is not the correct explanation of A.

UNSCRAMBLED WORDS

JANUARY 2023

1-g- MAXIMA	2-h- HYPERBOLA
3-i- SEQUENCE	4-j- DERANGEMENT
5-f- CHORD	6-e- EXPRESSION
7-d- FACTORISATION	8-c- DEGREE
9-b- PLANE	10-a- TRIANGLE

CASE-BASE QUESTIONS

1. (i) $c(x) = x^3 - 45x^2 + 600x$

$\therefore c'(x) = \frac{d}{dx}(c(x)) = 3x^2 - 90x + 600$

(ii) where, $c'(x) = 3x^2 - 90x + 600$

For $c'(x) = 0$

$\Rightarrow 3(x^2 - 30x + 200) = 0$

$\Rightarrow 3(x^2 - 20x - 10x + 200) = 0$

$\Rightarrow 3[x(x - 20) - 10(x - 20)] = 0$

$\Rightarrow (x - 10)(x - 20) = 0 \Rightarrow x = 10 \text{ or } 20$

Possible number of trees are 10 or 20.

(iii) $c'(x) = 3x^2 - 90x + 600$

$c''(x) = 6x - 90$

$\therefore c'(x) = 0 \Rightarrow x = 10, 20$

$c''(10) = 60 - 90 = -30 < 0$

$c''(20) = 6(20) - 90 = 120 - 90 = 30 > 0$

Thus, they must place order for 20 trees.

OR

At $x = 20$, $c(x) = (20)^3 - 45(20)^2 + 600(20)$

$= 8000 - 18000 + 12000 = ₹ 2000$

The least amount spend by them is ₹ 2000.

2. (i) We have, $\frac{x^2}{36} + \frac{y^2}{25} = 1$

$\Rightarrow y^2 = \frac{25}{36}(36 - x^2) \Rightarrow y = \pm \frac{5}{6}\sqrt{36 - x^2}$

$\text{Ar}(AOCBA) = \frac{5}{6} \int_{-6}^6 \sqrt{36 - x^2} dx = \frac{5}{6} \cdot 2 \int_0^6 \sqrt{36 - x^2} dx$

$= \frac{5}{3} \left[\frac{x}{2} \sqrt{36 - x^2} + 18 \sin^{-1} \frac{x}{6} \right]_0^6 = \frac{5}{3} \left[0 + 18 \cdot \frac{\pi}{2} - (0 + 0) \right]$

$= \frac{5}{3}(9\pi) = 15\pi \text{ sq. units}$

(ii) Area of whole shape of an ice-cream = Area of region $ABCD = 2(\text{ar}(AOCBA)) = 2 \times 15\pi = 30\pi \text{ sq units.}$

3. Let M : The applications in modern dance.

C : The applications in contemporary dance.

A : Age category is below 15.

B : Age category is 15 or above.

Dance form	Modern	Contemporary
Applications	960	1040
Below 15	192	936
15 or above	768	104

(i) $P\left(\frac{B}{C}\right) = \frac{104}{1040} = \frac{1}{10} = 0.1$

(ii) $P\left(\frac{M}{A}\right) = \frac{P(M) \cdot P(A/M)}{P(M) \cdot P(A/M) + P(C) \cdot P(A/C)}$

$$= \frac{\frac{960}{2000} \cdot \frac{192}{960}}{\frac{960}{2000} \cdot \frac{192}{960} + \frac{1040}{2000} \cdot \frac{936}{1040}}$$

$$= \frac{192}{192 + 936} = \frac{48}{48 + 234} = \frac{8}{8 + 39} = \frac{8}{47}$$

4. Let x be the length and y be the breadth of rectangular plot of land.

Given $(x - 50)(y + 50) = xy$

$\Rightarrow xy + 50x - 50y - 2500 = xy$

$\Rightarrow x - y = 50$... (1)

Also, $(x - 10)(y - 20) = xy - 5300$

$\Rightarrow xy - 20x - 10y + 200 = xy - 5300$

$\Rightarrow 2x + y = 550$... (2)

(i) Required system of equation is

$x - y = 50 ; 2x + y = 550.$

(ii) We have, $x - y = 50$ and $2x + y = 550$

$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 50 \\ 550 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore AX = B$

$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix},$

which is required matrix equation

(iii) Solving (1) and (2), we get

$x = 200, y = 150$

$\therefore$ Length and breadth of rectangular plot of land is 200 m, 150 m respectively.

OR

Area of rectangular plot = xy

$= 200 \text{ m} \times 150 \text{ m} = 30000 \text{ m}^2.$

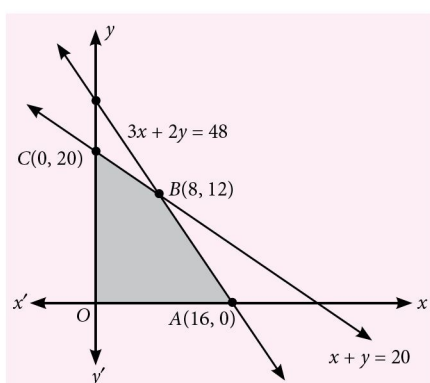
5. (i) R_3 is a reflexive and transitive relation but not symmetric as $(1, 2) \in R_3$ but $(2, 1) \notin R_3$

(ii) R_4 is a reflexive and symmetric relation but not a transitive as $(3, 2)$ and $(2, 1) \in R_4$ but $(3, 1) \notin R_4$.

6. (i) Objective function, $z = 22x + 18y$ subject to constraint, $x + y \leq 20, 3x + 2y \leq 48 ; x, y \geq 0$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (b) 2. (c) 3. (d) 4. (c) 5. (b)
 6. (c) 7. (a, c, d) 8. (a, b, c, d) 9. (b, d) 10. (a, b, c)
 11. (a, c) 12. (a, b, c) 13. (a, b) 14. (b) 15. (d)
 16. (a) 17. (314) 18. (4) 19. (9) 20. (6)



∴ The number of corner points is 4.

(ii) The value of z at corner points are 0, 352, 392 and 360.

Required sum = $0 + 352 + 392 + 360 = 1104$

(iii) The coordinates of corner points A, B, C, O are (16, 0), (8, 12), (0, 20) and (0, 0)

At A(0, 0), $Z = 0$; At A(16, 0), $Z = 352$;

At B(8, 12), $Z = 392$, At C(0, 20), $Z = 360$

To get the maximum profit, electric cars must be purchased by the dealer.

OR

To get maximum profit 8 diesel cars must be purchased by dealer.

7. (i) The line along which motorcycle A is running, is $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$$

$$\Rightarrow x = \lambda, y = 2\lambda, z = -\lambda \Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

(ii) Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$,

$$\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k} \therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$$

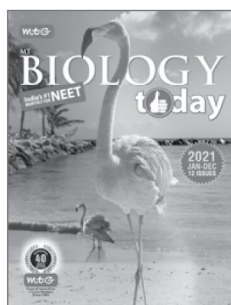
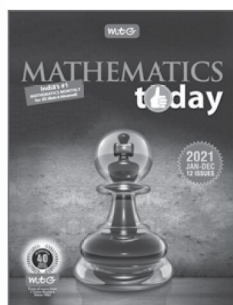
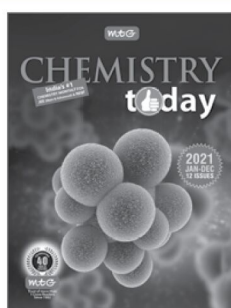
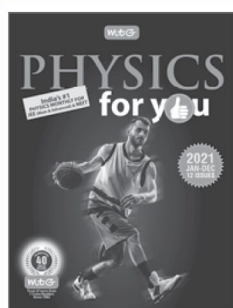
$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\text{Now, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k}) \\ = 9 - 9 = 0$$

Hence, shortest distance between the given lines is 0.



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Brain Teaser

LOGICAL REASONING

for Various Competitive Exams

1. Find the wrong term in the given series.
AN36, BO25, CP16, DS9, ER4
(a) AN36 (b) BO25 (c) CP16 (d) DS9

2. Find the missing term in the given series
51975, 9450, 2100, 600, 240, 160, ?
(a) 80 (b) 120 (c) 320 (d) 240

3. There is a certain relationship between the pair of words on the either side of :: Identify the relationship and find the missing word.

Genuine : Authentic :: Mirage : ?

- (a) Reflection (b) Hideout
(c) Illusion (d) Image

4. Three of the following four have similar relationships and hence form a group. Which one does not belong to the group?

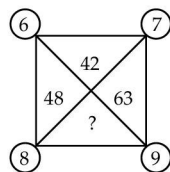
- (a) SAFETY : VYICWW
(b) SMOKER : VKRIHP
(c) SERIES : VCUGHQ
(d) HEALTH : KCYJYF

5. Find the missing term, if the given matrix follows a certain rule row wise or column wise.

A 2	C 5	F 9	J 14
C 4	F 8	J 13	O 19
E 6	I 11	N 17	T 24
G 8	L 14	R 21	?

- (a) U 20 (b) Z 21 (c) Y 29 (d) U 30

6. Find the missing character, if a certain pattern is followed for the values.



- (a) 64 (b) 81 (c) 72 (d) 144

Direction Q.(7 and 8): Questions below based upon codes given to a word :

P L E A S U R E
△ □ ○ ◇ ◇ △ ▽ ○

Under each question a word or a group of letters or numbers are given. Find the correct code for them. Mark the correct answers from the choices.

7. What is the code for PURSE?

- (a) △ □ ▽ ◇ ○ (b) ◇ ○ ◇ □ △
(c) △ ▽ △ ○ ◇ (d) ○ △ □ △ ▽

8. Which word stands for ◇ ◇ □ ○ ?

- (a) P L U S (b) L E A P
(c) S A L E (d) R A S P

Direction Q.(9 and 10): Read the following information carefully and answer the questions given below:

Ashok Mehta has three children, Usha, Ramchander and Sunil, Sunil married Rita, the eldest daughter of Mr. and Mrs. Mathur. The Mathurs married their youngest daughter to the eldest son of Mr. and Mrs. Saxena and they had two children named Sanjay and Sunita. The Mathur have two more children, Rakesh and Bindu, both elder to Shani. Sonu and Surinder are sons of Sunil and Rita. Lata is the daughter of Sanjay.

9. What is the surname of Lata?

- (a) Saxena (b) Mathur
(c) Sanjay (d) Mehta

10. How is Sonu related to the father of Rita?

- (a) Grandson (b) Son-in-law
(c) Son (d) Cousin

11. Bharat is facing the direction which is 135° to the clockwise of north. He starts turning and follows the following sequence:

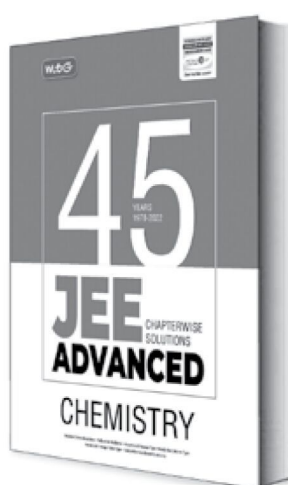
31° anticlockwise, 43° clockwise, 67° anticlockwise, 83° clockwise, 109° anticlockwise, 113° anti-clockwise, 127° clockwise, 143° clockwise, 159° anti-clockwise in the same order. Which direction is he facing now?

- (a) North-West (b) North-East
(c) South (d) South-West

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F _ O _ U _ N _ D

(a) <, <, >, = (b) <, =, =, <
(c) ≥, =, =, ≥ (d) >, >, =, <

- II. All rectangles are circles.

- (a) Lawyer, Teacher (b) Manager, Teacher
(c) Doctor, Manager (d) Lawyer, Manager

$$\begin{array}{cccccc} \downarrow^{+3} & \downarrow^{-2} & \downarrow^{-2} & \downarrow^{-2} & \downarrow^{+5} & \downarrow^{-2} \\ \text{K} & \text{C} & \text{Y} & \text{I} & \text{Y} & \text{F} \end{array}$$

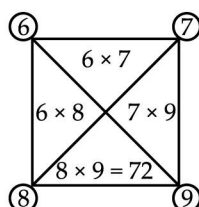
5. (c) : Column 1 : A 2 $\Rightarrow$ C 4 $\Rightarrow$ E 6 $\Rightarrow$ G 8

Column 2 : C 5 $\Rightarrow$ F 8 $\Rightarrow$ I 11 $\Rightarrow$ L 14

Column 3 : F 9 $\Rightarrow$ J 13 $\Rightarrow$ N 17 $\Rightarrow$ R 21

Column 4 : J 14 $\Rightarrow$ O 19 $\Rightarrow$ T 24 $\Rightarrow$ Y 29

6. (c) :



7. (a) : PURSE $\rightarrow$ $\triangle$ $\square$ $\cup$ $\diamond$ $\circ$

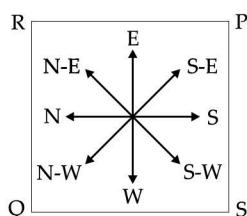
8. (c) : $\diamond$ $\square$ $\square$ $\square$ $\square$ $\rightarrow$ SALE

9. (a) : Lata Saxena

10. (a) : Sonu is the grandson of Mr. Mathur.

11. (b) : $-31^\circ + 43^\circ - 67^\circ + 83^\circ - 109^\circ - 113^\circ + 127^\circ + 143^\circ - 159^\circ = -83^\circ$.

Initial (South-East) -83° = Final point (North-East).



12. (b) : $F < O = U = N < D$

Now, $F < N$ and $U < D$

13. (c) : D and W are opposite to each other. So, option (a) and (d) are ruled out. As D and W has to be opposite the '?' should be replaced by Y which will make Y and U common in figure III and IV.

14. (c) :

Consonant	Vowel	Number
-----------	-------	--------

Such combinations are:

T E 3

 ;

K A 8

 ;

W I 6

Thus, there are three such Vowels.

15. (a) :

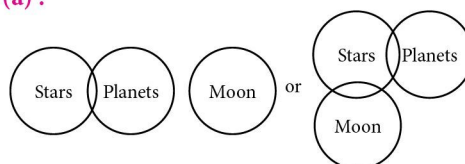
Symbol	Consonant	Consonant
--------	-----------	-----------

Such combinations is:

\$ V N

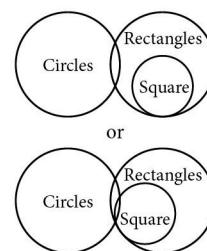
Thus, there is one such consonants.

16. (a) :



Stars may or may not be moon thus either of the conclusion (I) or (II) is valid. Thus, either conclusion (I) or (II) follows.

17. (d) : Square may or may not be circle thus conclusion (I) is not valid. All rectangle may or may not be circle thus conclusion (II) is not valid. Thus none of the conclusion follows.



(18-20) : As, two person of same profession does not stay on same floor and there are 3 managers, so on each floor there will be one managers., So we have the following situation :

	Teacher	Manager	Doctor			IV
	Lawyer		Teacher		Manager	III
		Lawyer		Manager		II

18. (b)

19. (a)

20. (d)



EXAM ALERT 2023

Exam	Date
JEE Main Session 2	6 th to 12 th April
WB JEE	30 th April
CUET	21 st to 31 st May
MHT CET PCM	9 th to 13 th May
CBSE Class 12 th	
CBSE Chemistry	28 th Feb
CBSE Physics	6 th March
CBSE Mathematics	11 th March
CBSE Biology	16 th March

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Duration : 60 minutes

SECTION-I

Single Option Correct Type

- Which of the following numbers are rational?
(a) $\sin 15^\circ$ (b) $\cos 15^\circ$
(c) $\sin 15^\circ \cos 15^\circ$ (d) $\sin 15^\circ \cos 75^\circ$
- z is a complex number such that $z \neq 1$, $|z|=1$. The real part of $\frac{z-1}{z+1}$ is
(a) $\frac{1}{|z+1|^2}$ (b) $\frac{-1}{|z+1|^2}$ (c) $\frac{\sqrt{2}}{|z+1|^2}$ (d) 0
- The number of ways of selecting 10 objects from 30 objects, of which 10 are alike and the rest are all different is
(a) $2^{20} + \binom{20}{10}$ (b) $2^{20} - \binom{20}{9}$
(c) $2^{19} + \binom{19}{9}$ (d) $2^{19} - \binom{20}{10}$
- The term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ is
(a) $\frac{17}{27}$ (b) $\frac{17}{54}$ (c) $\frac{19}{27}$ (d) $\frac{19}{54}$
- The interior angles of a convex polygon are in A.P. The smallest angle is 120° and the common difference is 5° . The number of its sides is
(a) 9 (b) 10 (c) 13 (d) 16
- $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x =$
(a) e^4 (b) e^3 (c) e^2 (d) e
- If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$, then angle between vector $\vec{b}$ and $\vec{c}$ is

- (a) 60° (b) 30° (c) 45° (d) 90°

- The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is
(a) 0 (b) 1 (c) 4 (d) 2

- Let $g : (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c$, for all $x > 0$, where c is an arbitrary constant. Then

- (a) g is decreasing in $\left(0, \frac{\pi}{4}\right)$
(b) g' is increasing in $\left(0, \frac{\pi}{4}\right)$
(c) $g + g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$
(d) $g - g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

- The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is
(a) 1 (b) 4 (c) 6 (d) 8

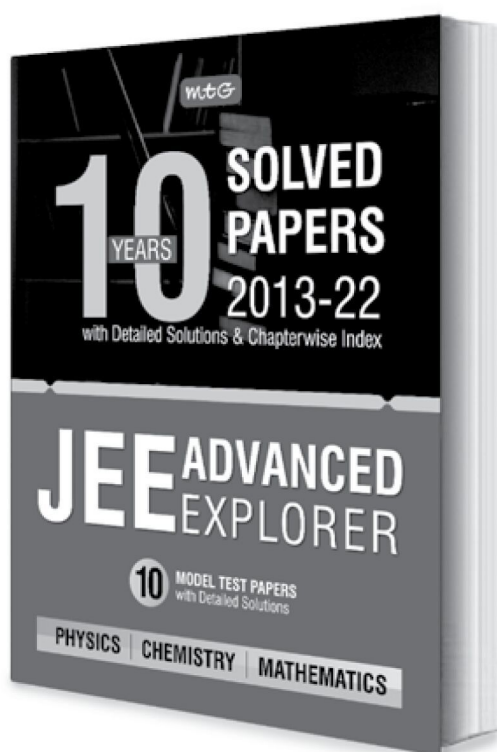
SECTION-II

Numerical Answer Type

- A circle passes through the points (4, 1) and (6, 5) and has its centre on the line $x - y = 8$. Its radius is _____.
- If $\lim_{n \rightarrow \infty} \frac{(n+1)^{k-1}}{n^{k+1}} [(nk+1) + (nk+2) + \dots + (nk+n)] = 33 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^{k+1}} [1^k + 2^k + 3^k + \dots + n^k]$, then the integral value of k is equal to _____.



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13. If $\int \frac{dx}{(x^2+x+1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$,

$x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}a+b)$ is equal to _____.

14. $\lim_{n \rightarrow \infty} \frac{1}{2^n} \left(\frac{1}{\sqrt{1-\frac{1}{2^n}}} + \frac{1}{\sqrt{1-\frac{2}{2^n}}} + \frac{1}{\sqrt{1-\frac{3}{2^n}}} + \dots + \frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}} \right)$

is equal to _____.

15. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____.

SOLUTIONS

1. (c) : $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin 15^\circ \cos 15^\circ = \frac{1}{2} \sin 30^\circ = \frac{1}{4}$

$\sin 15^\circ \cos 75^\circ = \sin^2 15^\circ = \frac{4-2\sqrt{3}}{8}$

2. (d) : $|z|=1 \Rightarrow z \bar{z}=1$, $\bar{z} = \frac{1}{z}$ and $\operatorname{Re} z = \frac{1}{2}(z + \bar{z})$

$\therefore \operatorname{Re}\left(\frac{z-1}{z+1}\right) = \frac{1}{2}\left(\frac{z-1}{z+1} + \frac{\bar{z}-1}{\bar{z}+1}\right) = \frac{1}{2}\left(\frac{z-1}{z+1} + \frac{\bar{z}-1}{\bar{z}+1}\right)$

$= \frac{1}{2}\left(\frac{z-1}{z+1} + \frac{\frac{1}{z}-1}{\frac{1}{z}+1}\right) = \frac{1}{2}\left(\frac{z-1}{z+1} + \frac{1-z}{z+1}\right) = 0$

3. (c) : Let the objects be $aaa \dots ab_1b_2b_3 \dots b_{20}$

We can choose r a 's in one way and $(10-r)$ b 's in $\binom{20}{10-r}$ ways

$\therefore$ The desired number is

$= \sum_{r=0}^{10} \binom{20}{10-r} = \binom{20}{10} + \binom{20}{9} + \dots + \binom{20}{0}$

$= \frac{1}{2} \left[\binom{20}{0} + \binom{20}{1} + \dots + \binom{20}{10} + \binom{20}{10} + \binom{20}{11} + \dots + \binom{20}{20} \right]$

$= \frac{1}{2} \cdot \left(2^{20} + \binom{20}{10} \right) \left(\text{since } \binom{20}{r} = \binom{20}{20-r} \right)$

$= 2^{19} + \frac{1}{2} \binom{20}{10} = 2^{19} + \binom{19}{9}$

4. (b) : $t_{r+1} \ln \left(\frac{3x^2}{2} - \frac{1}{3x} \right)^9 = \binom{9}{r} \left(\frac{3x^2}{2} \right)^{9-r} \left(-\frac{1}{3x} \right)^r$

$= \binom{9}{r} \left(\frac{3}{2} \right)^{9-r} \left(-\frac{1}{3} \right)^r \cdot x^{18-3r}$

The constant in the given expression is

$= 1 \cdot \binom{9}{6} \left(\frac{3}{2} \right)^3 \left(-\frac{1}{3} \right)^6 + 2 \cdot \binom{9}{7} \left(\frac{3}{2} \right)^2 \left(-\frac{1}{3} \right)^7 = \frac{7}{18} - \frac{2}{27} = \frac{17}{54}$

5. (a) : If n is the number of its sides, the sum of its interior angles is

$\frac{n}{2} [2 \times 120^\circ + (n-1)5^\circ] = \frac{5n}{2} [n+47] \dots(i)$

But the sum of the interior angles is

$180^\circ n - 360^\circ = 180(n-2) \dots(ii)$

From (i) and (ii), $\frac{5n}{2} (n+47) = 180(n-2)$

$\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow n = 9, 16$

In a convex polygon no angle exceeds 180° .

$\therefore n \neq 16$ and $n = 9$.

6. (a) : $\lim_{x \rightarrow \infty} \left(\frac{x^2+5x+3}{x^2+x+3} \right)^x$ (1^∞ form)

$= \exp \lim_{x \rightarrow \infty} \left(\frac{x^2+5x+3}{x^2+x+3} - 1 \right) x = \exp \lim_{x \rightarrow \infty} \frac{4x^2}{x^2+x+3} = e^4$

7. (a) : Given $\vec{a} + \vec{b} + \vec{c} = 0$, we need angle between $\vec{b}$ and $\vec{c}$, so consider $\vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow \vec{b}^2 + \vec{c}^2 + 2|\vec{b}||\vec{c}|\cos\theta = \vec{a}^2$

$\Rightarrow \cos\theta = \frac{\vec{a}^2 - \vec{b}^2 - \vec{c}^2}{2|\vec{b}||\vec{c}|} = \frac{49-25-9}{2 \times 5 \times 3} = \frac{1}{2} \therefore \theta = 60^\circ$

8. (d) : Let a, b, c, d, e be five observations.

$\therefore \text{S.D.} = \sqrt{\frac{\sum(a-\bar{x})^2}{5}} = 0 \Rightarrow \sum(a-\bar{x})^2 = 0$

$\Rightarrow (a-\bar{x})^2 + (b-\bar{x})^2 + (c-\bar{x})^2 + (d-\bar{x})^2 + (e-\bar{x})^2 = 0$

$\Rightarrow a-\bar{x} = b-\bar{x} = c-\bar{x} = d-\bar{x} = e-\bar{x} = 0$

$\Rightarrow a = b = c = d = e = \bar{x} \Rightarrow a = b = c = d = e = 9$ $[\because \bar{x} = 9]$

$\therefore a + b + c + d + e = 9 \times 5 = 45$

Let x be the change in term d . Then,

New sum $= a + b + c + (d+x) + e = 5 \times 10 = 50 \Rightarrow x = 5$

$\therefore$ New observations becomes, 9, 9, 9, 14, 9

New S.D. $= \sqrt{\frac{\sum(a-\bar{x})^2}{5}} = \sqrt{\frac{1^2+1^2+1^2+4^2+1^2}{5}} = \sqrt{\frac{20}{5}} = 2$

9. (d): We have,

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + C$$

Differentiate both sides as,

$$\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} = \frac{d}{dx} \left[\frac{xg(x)}{e^x + 1} + C \right]$$

We get, $g'(x) = \cos x - \sin x$

$$g(x) = \sin x + \cos x \quad \therefore g - g' = 2 \sin x$$

We know $\sin x$ is in increasing in $\left(0, \frac{\pi}{2}\right)$.

So, $g - g'$ will be increasing in $\left(0, \frac{\pi}{2}\right)$.

10. (d): $(11)^{1011} + (1011)^{11} = (9 + 2)^{1011} + (1008 + 3)^{11}$

When this is divided by 9, we are left with

$$2^{1011} + 3^{11} = (8)^{337} + 3^{11} = (9 - 1)^{337} + 3^{11}$$

= multiple of 9 - 1 + 9^{5.3}

$$\text{Remainder} = -1 + 9 = 8$$

11. (5): The centre is $(\alpha, \alpha - 8)$ and

$$r^2 = (\alpha - 4)^2 + (\alpha - 9)^2 = (\alpha - 6)^2 + (\alpha - 13)^2$$

$$\Rightarrow 12\alpha = 108, \alpha = 9, r^2 = 5^2 \Rightarrow r = 5$$

12. (5): We have,

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{(n+1)^{k-1}}{n^{k+1}} [(nk+1) + (nk+2) + \dots + (nk+n)] \\ = 33 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^{k+1}} [1^k + 2^k + 3^k + \dots + n^k] \end{aligned}$$

$$\text{Consider L.H.S., } \lim_{n \rightarrow \infty} \frac{(n+1)^{k-1}}{n^{k+1}} \left[n^2 k + \frac{n(n+1)}{2} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(n+1)^{k-1}}{n^{k+1}} n^2 \left[k + \frac{\left(1 + \frac{1}{n}\right)}{2} \right]$$

$$= \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^{k-1} \left[k + \frac{\left(1 + \frac{1}{n}\right)}{2} \right]$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{k-1} \left[k + \frac{\left(1 + \frac{1}{n}\right)}{2} \right] = k + \frac{1}{2}$$

Now consider R.H.S.,

$$33 \lim_{n \rightarrow \infty} \frac{1}{n^{k+1}} [1^k + 2^k + 3^k + \dots + n^k] = 33 \left(\frac{1}{k+1} \right)$$

Given, L.H.S. = R.H.S.

$$\therefore k + \frac{1}{2} = 33 \left(\frac{1}{k+1} \right) \Rightarrow k = 5, \frac{-13}{2}$$

$$13. (15): \text{ Let } I = \int \frac{dx}{(x^2 + x + 1)^2} = \int \frac{dx}{\left[\left(x + \frac{1}{2} \right)^2 + \frac{3}{4} \right]^2}$$

$$\Rightarrow I = 16 \int \frac{dx}{[(2x+1)^2 + 3]^2}$$

$$\text{Put } 2x+1 = \sqrt{3} \tan \theta \Rightarrow \theta = \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)$$

$$\Rightarrow 2dx = \sqrt{3} \sec^2 \theta d\theta$$

$$\therefore I = \frac{16}{2} \int \frac{\sqrt{3} \sec^2 \theta d\theta}{(3 \tan^2 \theta + 3)^2} = \frac{8\sqrt{3}}{9} \int \frac{\sec^2 \theta}{(\tan^2 \theta + 1)^2} d\theta$$

$$= \frac{8}{3\sqrt{3}} \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \frac{8}{3\sqrt{3}} \int \cos^2 \theta d\theta = \frac{8}{3\sqrt{3}} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{4}{3\sqrt{3}} \left[\theta + \frac{\sin 2\theta}{2} \right] = \frac{4}{3\sqrt{3}} \left[\tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right] + \frac{2}{3\sqrt{3}} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$= \frac{4}{3\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + \frac{1}{3} \left(\frac{2x+1}{x^2 + x + 1} \right)$$

$$\Rightarrow a = \frac{4}{3\sqrt{3}} \text{ and } b = \frac{1}{3} \therefore 9(\sqrt{3}a + b) = 9 \left(\frac{4}{3} + \frac{1}{3} \right) = 15$$

14. (2):

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} \left(\frac{1}{\sqrt{1 - \frac{1}{2^n}}} + \frac{1}{\sqrt{1 - \frac{2}{2^n}}} + \frac{1}{\sqrt{1 - \frac{3}{2^n}}} + \dots + \frac{1}{\sqrt{1 - \frac{2^n - 1}{2^n}}} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2^n} \sum_{x=1}^{2^n-1} \frac{1}{\sqrt{1 - \frac{x}{2^n}}}$$

$$\text{Sum of series} = \int_0^1 \frac{1}{\sqrt{1-x}} dx$$

$$= [-2\sqrt{1-x}]_0^1 = [-2(0) + 2(1)] = 2$$

15. (3): We have, $2\sin^2 \theta - \cos 2\theta = 0$

$$\Rightarrow 1 - 2\cos 2\theta = 0 \Rightarrow \cos 2\theta = 1/2$$

$$\Rightarrow 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{And } 2\cos^2 \theta + 3\sin \theta = 0 \Rightarrow 2 - 2\sin^2 \theta + 3\sin \theta = 0$$

$$\Rightarrow 2\sin^2 \theta - 3\sin \theta - 2 = 0 \Rightarrow 2\sin^2 \theta - 4\sin \theta + \sin \theta - 2 = 0$$

$$\Rightarrow (2\sin \theta + 1)(\sin \theta - 2) = 0$$

$$\Rightarrow \sin \theta = \frac{-1}{2} \quad [\because \sin \theta \neq 2]$$

$$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\therefore \text{Sum of solutions} = \frac{7\pi}{6} + \frac{11\pi}{6} = 3\pi \therefore k = 3$$

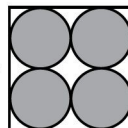




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- A and B are two positive integers such that $AB = 64$. Which of the following cannot be the value of $A + B$?
(a) 34 (b) 65 (c) 20 (d) 35
- Naresh's salary is 40% more than Rajesh's salary. How much percentage does Rajesh's salary less than Naresh's salary?
(a) $26\frac{5}{7}\%$ (b) $28\frac{4}{7}\%$ (c) $29\frac{1}{7}\%$ (d) $27\frac{3}{7}\%$
- ₹ 20000 is being compounded at 20% per annum. If the rate of interest is charged half yearly. What will be the amount after 2 years?
(a) ₹ 28292 (b) ₹ 27292 (c) ₹ 29282 (d) ₹ 22358
- By selling 36 mangoes, a vendor loses the selling price of 4 mangoes. His loss percent is
(a) $11\frac{2}{5}\%$ (b) $12\frac{3}{4}\%$
(c) 10% (d) None of these
- A and B started a business investing amounts of ₹ 50000 and ₹ 250000 respectively for a year. What will be B's share in the profit of ₹ 160000?
(a) ₹ 100000 (b) ₹ 60000
(c) ₹ 80000 (d) None of these
- The average of the 9 consecutive positive integers is 63. The product of the largest and smallest integers is
(a) 3935 (b) 3953
(c) 3853 (d) 3835
- A mixture with a certain quantity of milk contains 28 litres of water is worth ₹ 15 per litre. If pure milk be worth ₹ 21 per litre, how much milk is there in the mixture?
(a) 82 L (b) 75 L (c) 65 L (d) 70 L
- Three typists P, Q and R have to type 368 pages. P types one page in 8 minutes, Q in 18 minutes and R in 24 minutes. In what time will these pages be typed if they work together?
(a) 25 hours (b) 27.6 hours
(c) 27 hours (d) 28 hours
- A superfast train crosses a passenger train in 20 seconds. The speed of superfast train is 72 km/hr and speed of passenger train is 27 km/hr. Also the length of superfast train is 150 m, then find the length of the passenger train if they are moving in the same direction.
(a) 125 m (b) 100 m (c) 150 m (d) 200 m
- A yearly payment to the servant is ₹ 90 plus one turban. The servant leaves the job after 9 months and receives ₹ 65 and a turban. Then find the price of the turban is
(a) ₹ 10 (b) ₹ 15 (c) ₹ 7.50 (d) ₹ 5
- Four identical coins are placed in a square. For each coin the ratio of area to circumference is same as the ratio of circumference to area. Then the area of the square that is not covered by the coins is
(a) $16(\pi - 1)$ sq. units (b) $16(8 - \pi)$ sq. units
(c) $16(4 - \pi)$ sq. units (d) $16\left(4 - \frac{\pi}{2}\right)$ sq. units
- In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?
(a) $5! \times 3!$ (b) $3! \times 3!$
(c) $\frac{3! \times 3!}{2!}$ (d) $\frac{4! \times 3!}{2!}$
- A card is drawn from a deck of 52 cards. Find the probability of getting a red card or a heart or a king.
(a) $\frac{6}{13}$ (b) $\frac{7}{13}$ (c) $\frac{11}{26}$ (d) $\frac{15}{26}$



15. In a restaurant, 30 people have Idlies, 45 have dosas, 18 have both and 24 have neither. How many people visited the restaurant if only idlies and dosas are served in the restaurant?

- (a) 57 (b) 65 (c) 42 (d) 81

16. The least perfect square number which is divisible by 3, 4, 5, 6 and 8 is

- (a) 900 (b) 1200 (c) 2500 (d) 3600

17. Out of the total production of iron from haematite, an ore of iron, 20% of the ore gets wasted. Out of the remaining iron, only 25% is pure iron. If the pure iron obtained in a year from a mine of haematite was 80000 kg, then the quantity of haematite mined in the year is

- (a) 400000 kg (b) 500000 kg
(c) 450000 kg (d) 600000 kg

18. The difference between compound interest and simple interest at the same rate on ₹ 5000 for 2 years is ₹ 72. The rate of interest per annum is

- (a) 6% (b) 8% (c) 10% (d) 12%

19. A machine is sold at a profit of 10%. Had it been sold for ₹ 40 less, there would have been a loss of 10%. What was the cost price?

- (a) ₹ 175 (b) ₹ 200 (c) ₹ 225 (d) ₹ 150

20. Ratio of Ashok's age to Pradeep's age is 4 : 3. Ashok will be 26 years old after 6 years. How old is Pradeep now?

- (a) 18 years (b) 21 years (c) 15 years (d) 24 years

SOLUTIONS

1. (c) : $\frac{5}{8}$ of 4012.33 + $\frac{7}{10}$ of 3412.22

Taking approximate values,

$$= \frac{5}{8} \times 4012 + \frac{7}{10} \times 3412 = 5 \times 501.5 + 2388.4 \\ = 2507.5 + 2388.4 = 4895.9 \approx 4896$$

2. (d) : $AB = 64$

(A, B) may be any of the following

Case	Sum
1×64	$1 + 64 = 65$
2×32	$2 + 32 = 34$
4×16	$4 + 16 = 20$
8×8	$8 + 8 = 16$

Hence, $A + B$ cannot be 35.

3. (b) : Let Rajesh's salary be ₹ 100.

∴ Naresh's salary will be = $100 + 40\%$ of $100 = ₹ 140$.

∴ Rajesh gets ₹ 40 less than Naresh's salary.

$$\therefore \text{Required percentage} = \frac{40}{140} \times 100 = 28\frac{4}{7}\%$$

4. (c) : $r = 20\%$, $t = 2$ years = 4 half year

Rate is changed half yearly, so $r = \frac{20\%}{2} = 10\%$

$$\therefore A = 20000 \left(1 + \frac{20}{2(100)} \right)^4 = ₹ 29282$$

5. (c) : $CP_{36} - SP_{36} = SP_4$

$$CP_{36} = SP_{40}$$

Let the CP of each mango be ₹ 1

CP of 40 mangoes = ₹ 40

SP of 40 mangoes = ₹ 36

$$\text{Loss \%} = \left(\frac{4}{40} \times 100 \right) \% = 10\%$$

6. (d) : $\frac{A \text{ 'Share}}{B \text{ 'Share}} = \frac{50000}{250000} = 1:5$

Profit will be in the same ratio as in investment

$$\text{Now } 1x + 5x = 160000 \Rightarrow 6x = 160000$$

$$\Rightarrow x = \frac{160000}{6}; B \text{ 'Share} = \frac{5}{6} \times 160000 = ₹ 133333.3$$

7. (b) : When odd no. of integers are given,

Average is the middle no.

Hence, 5th number is the average, which is 63

Smallest Integer = $63 - 4 = 59$

Largest Integer = $63 + 4 = 67$

So, product is $59 \times 67 = 3953$

8. (d) : Let x L of milk is present in the mixture

$$\text{So, } (x + 28) \times 15 = 21x \Rightarrow 28 \times 15 = 21x - 15x$$

$$\Rightarrow x = \frac{28 \times 15}{6} = 70$$

Hence, 70 L of milk is present in the mixture.

9. (b) : L.C.M of 8, 18, 24 is 72

Hence in 72 minutes

P types 9 pages

Q types 4 pages

R types 3 pages

If they work together they type 16 pages in 72 minutes.

$$\therefore 1 \text{ page in } \frac{72}{16} \text{ minutes.}$$

$$\therefore 368 \text{ pages in } \frac{72}{16} \times 368 = 1656 \text{ min or } 27.6 \text{ hrs.}$$

10. (b) : Let the length of passenger train be x m.
Relative speed $= (72 - 27) = 45$ km/hr

$$= 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/s}$$

$$\text{Crossing Time} = \frac{\text{Sum of length of the two trains}}{\text{Relative speed}}$$

$$\Rightarrow 20 = \frac{(150 + x)}{\frac{25}{2}}$$

$$\therefore x = 100 \text{ m}$$

11. (a) : Let the cost of the turban be T .

Hence, total payment for one year $= ₹ 90 + T$. So the payment for 9 months should be $3/4(90 + T)$. But this is equal to $(65 + T)$. Equating the two, we get $T = ₹ 10$.

12. (c) : Let R be the radius of each circle.

$$\text{Then } \frac{\pi R^2}{2\pi R} = \frac{2\pi R}{\pi R^2}, \text{ which implies that } \frac{R}{2} = \frac{2}{R},$$

$$\text{i.e. } R^2 = 4, \text{ i.e. } R = 2.$$

Then the side of the square is 8 unit. Thus, the area of the square is 64 sq. units, while the area covered by each coin is $\pi(2)^2 = 4\pi$ sq. units. Since there are four coins, the area covered by coins is $4(4\pi) = 16\pi$ sq. units. Thus, the area not covered by the coins is $64 - 16\pi = 16(4 - \pi)$ sq. units.

13. (d) : AAU B C S to be arranged in $4!$ ways

$$\text{AAU can be arranged in } \frac{3!}{2!} \text{ ways}$$

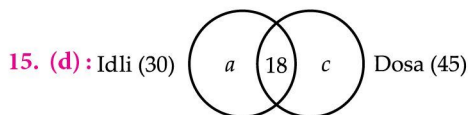
$$\text{Total ways} = \frac{3!}{2!} \times 4!$$

$$\textbf{14. (b) : } n(S) = {}^{52}C_1 = 52$$

Favourable number of outcomes

$$= 26 \text{ (red; includes heart)} + 2 \text{ (black kings)} = 28$$

$$P(\text{red or heart or king}) = \frac{28}{52} = \frac{7}{13}$$



$$a + 18 = 30 \Rightarrow a = 12$$

$$c + 18 = 45 \Rightarrow c = 27$$

$$\text{Total people} = a + c + 18 + 24 = 12 + 27 + 18 + 24 = 81$$

16. (d) : LCM of (3, 4, 5, 6, 8) = 120

But it is not a perfect square

$$120 = 2^3 \times 3^1 \times 5^1$$

So, it should be multiplied by $(2 \times 3 \times 5)$ to become a perfect square.

$$\text{So, required value} = 120 \times 30 = 3600$$

17. (a) : Let the quantity of haematite mined be x kg.

$$\text{Then, } x \times \frac{80}{100} \times \frac{25}{100} = 80000$$

$$\Rightarrow x = 80000 \times \frac{100}{80} \times \frac{100}{25} = 400000 \text{ kg.}$$

$$\textbf{18. (d) : } \text{Difference} = \frac{Pr^2}{100^2} \Rightarrow 72 = \frac{Pr^2}{100^2}$$

$$\Rightarrow r^2 = \frac{72 \times 100 \times 100}{5000} = 144$$

$$\therefore r = 12\%$$

19. (b) : Let the cost price of the machine be ₹ x .

$$\text{Then, } x \times \frac{110}{100} - 40 = x \times \frac{90}{100}$$

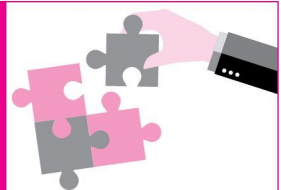
$$\Rightarrow x = 40 \times \frac{100}{20} = ₹ 200$$

20. (c) : Ashok's present age = $26 - 6 = 20$ years

$$\therefore \text{Pradeep's present age} = 20 \times \frac{3}{4} = 15 \text{ years}$$



PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6×6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

2÷		3÷	2÷	8+	2-
1-					
3-	6	40×			9+
	2-				
54×		3-		3-	
	1-		15+		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.



CBSE warm-up!

CLASS-XI

Practice questions for CBSE Exams as per the latest pattern
and reduced syllabus by CBSE for the academic session 2022-23.

Practice Paper 2022-23

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- (a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- (b) Section A has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each.
- (c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- (d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- (e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- (f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If there are 15 buses plying between places A and B, then number of ways, in which round trip can be made if the return journey was made on the same bus, is
(a) 15 (b) 210
(c) 225 (d) None of these
2. If $A = \{x : x \text{ is a positive factor of a prime number } p\}$, then roster form of A is
(a) $A = \{1, 2\}$ (b) $A = \{1, p\}$
(c) $A = \{1\}$ (d) $A = \{p\}$
3. If $A + B = 5\pi/4$, then value of $(1 + \tan A)(1 + \tan B)$ equals
(a) 1 (b) 2 (c) -2 (d) -1
4. If $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$, then $\frac{z_1}{z_2}$ is equal to
(a) $\frac{8}{5} - \frac{1}{5}i$ (b) $\frac{8}{5} + \frac{1}{5}i$
(c) $-\frac{8}{5} - \frac{1}{5}i$ (d) $-\frac{8}{5} + \frac{1}{5}i$
5. If $(3a + 5, 5 - b) = (8, -2)$, then
(a) $a = 1, b = 3$ (b) $a = 1, b = 4$
(c) $a = 1, b = 3/2$ (d) $a = 1, b = 7$
6. If $\frac{5-2x}{3} \leq \frac{x}{6} - 5$, then $x \in$
(a) $[2, \infty)$ (b) $[-8, 8]$ (c) $[4, \infty)$ (d) $[8, \infty)$
7. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows an even number". Then, E and F are
(a) mutually exclusive (b) exhaustive
(c) mutually exclusive and exhaustive
(d) None of these
8. If ${}^nC_2 - {}^nC_1 = 35$, then find the value of n.
(a) 10 (b) 11 (c) 12 (d) 13
9. The mean of a set of numbers is $\bar{x}$. If each item is decreased by 4, then mean of new observations (set) is
(a) Unaltered (b) Raised by 4 units in $\bar{x}$
(c) Reduced by 4 units in $\bar{x}$
(d) None of these

10. $\lim_{x \rightarrow 2} \left(\frac{x^2 - 1}{x^4 - 2x^3 + 4x - 5} \right) =$
 (a) 0 (b) 5 (c) 6 (d) 1
11. If $\cos \theta = 3/5$ and θ lies in fourth quadrant, then value of $\operatorname{cosec} \theta$ is
 (a) $-5/4$ (b) 2 (c) -2 (d) $-4/5$
12. Find the centre and radius of circle $x^2 + y^2 - 2x + 4y = 8$.
 (a) $(1, 2), \sqrt{13}$ (b) $(1, -2), \sqrt{13}$
 (c) $(-1, -2), \sqrt{13}$ (d) $(2, -1), 13$
13. Let the sequence a_n be defined as follows :
 $a_1 = 1, a_n = a_{n-1} + 2$ for $n \geq 2$,
 then the corresponding series is
 (a) $1 + 3 + 5 + 7 + 9 + \dots$
 (b) $1 + 5 + 9 + 13 + 17 + \dots$
 (c) $1 + 4 + 7 + 10 + 13 + \dots$
 (d) $1 + 6 + 11 + 16 + 21 + \dots$
14. The probability that a non-leap year selected at random will have 53 Sundays is
 (a) 0 (b) 1 (c) $1/7$ (d) $2/7$
15. Compute the derivative of $f(x) = \sin^2 x$.
 (a) $\sin 2x$ (b) $2 \sin x$
 (c) $2x \sin 2x$ (d) $\sin x$
16. Find the mean deviation about the mean for the following data : 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5
 (a) 8.2 (b) 9.2 (c) 6.2 (d) 5.2
17. Find the angle between the x -axis and the line joining the points $(3, -1)$ and $(4, -2)$.
 (a) 130° (b) 135°
 (c) 150° (d) None of these
18. Expansion of $\left(\frac{x}{3} + \frac{1}{x} \right)^5$ is equal to
 (a) $\frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$
 (b) $\frac{x^5}{420} + \frac{5x^4}{2} + \frac{3x^3}{81} + \frac{10x}{27} + \frac{5}{x} + \frac{2}{x^4}$
 (c) $\frac{2x^5}{243} + \frac{4x^4}{3} + \frac{3x^3}{81} + \frac{10x}{9} + \frac{10}{9x} + \frac{5}{3x^3}$
 (d) None of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. **Assertion (A):** The distance between the points $(1 + \sqrt{11}, 0, 0)$ and $(1, -2, 3)$ is $2\sqrt{6}$ units.

Reason (R): Distance between any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is,

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

20. **Assertion (A):** The derivative of $h(x) = \frac{x + \cos x}{\tan x}$ is $\frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{(\tan x)^2}$.

Reason (R): $\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{(v)^2}$.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the coordinates of the point on the line joining the points $P(3, -4)$ and $Q(-2, 5)$ that is twice as far from P as from Q .

OR

Prove that the points $A(1, 4)$, $B(3, -2)$ and $C(4, -5)$ are collinear.

22. Convert $\frac{1+7i}{(2-i)^2}$ into standard form.

23. Prove that $\tan 70^\circ = 2 \tan 50^\circ + \tan 20^\circ$.

OR

If $2 \cos \theta = a + \frac{1}{a}$, then find the value of $2 \cos 2\theta$.

24. Find the set of all values of x satisfying the inequality: $\frac{-2}{3} < 1 - \frac{x}{3} \leq \frac{2}{3}$

25. Using Venn diagram, prove that $(A - B) \cup (A \cap B) = A$.

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Find n , if $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5 : 3$.

27. Without expanding find the value of $(2x - 1)^4 + 4(2x - 1)^3(3 - 2x) + 6(2x - 1)^2(3 - 2x)^2 + 4(2x - 1)(3 - 2x)^3 + (3 - 2x)^4$

OR

Find the value of $(1.02)^6$, correct upto four places of decimals by using binomial theorem.

28. Find the equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directrices.

OR

Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, length of latus rectum is 5 and the centre is $(0, 0)$ and axes as coordinate axes.

29. If $\tan \beta = \frac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha}$, then show that

$$\tan(\alpha - \beta) = (1 - n) \tan \alpha.$$

OR

Prove that : $\sin x + \sin y + \sin z - \sin(x + y + z)$
 $= 4 \sin \frac{x+y}{2} \sin \frac{y+z}{2} \sin \frac{x+z}{2}.$

30. Three dice are thrown simultaneously. Find the probability that
 (i) all of them show the same face.
 (ii) all show distinct faces.
31. If $A = \{2, 3\}$, $B = \{6, 8\}$, $C = \{1, 2\}$ and $D = \{6, 9\}$, then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. If A , B and C be sets. Then show that
 (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
33. Compute variance and standard deviation for the following data.

Class interval	Frequency
45-55	7
55-65	20
65-75	27
75-85	23
85-95	13
95-105	6
105-115	3
115-125	1

OR

Determine the mean and standard deviation for the following distribution:

Marks	Frequency
2	1
3	6
4	6
5	8
6	8
7	2
8	2
9	3
10	0
11	2
12	1
13	0
14	0
15	0
16	1

34. If S_1 , S_2 and S_3 be respectively the sum of n , $2n$ and $3n$ terms of a G.P. then prove that
 $S_1(S_3 - S_2) = (S_2 - S_1)^2.$
35. Find the derivative of $\cos(x^2 + 1)$ w.r.t. x from the first principle.

OR

Evaluate : $\lim_{x \rightarrow \sqrt{10}} \frac{\sqrt{7-2x} - (\sqrt{5} - \sqrt{2})}{x^2 - 10}$

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case-study 1:** Read the following passage and answer the questions given below.
 Hina's house is situated at $A(1, 2, 3)$. She was going to her school. First she went to her friends home located at $B(3, 3, 5)$. Her school located at $C(3, 6, 5)$. Let D be the position of park.



(D)



(C)



(A)



(B)

- (i) What is the distance between Hina's house and her friend's home?
- (ii) Calculate the total distance covered by Hina.
- (iii) Find the shortest distance between Hina's home to her school.

OR

Find the locus of park which is equidistant from $A(1, 2, 3)$ and $B(3, 3, 5)$.

- 37. Case-study 2 :** Read the following passage and answer the questions given below.

On sunday, Riyan and Ravi decided to play with cards. Riyan found a deck of cards at his home but he observed that 1 card is missing in 52 cards. After checking on cards he found that the missing card is an ace of spade. After that they start playing with the remaining cards and Riyan asked Ravi to choose 4 cards from the 51 cards.



- (i) Find the number of ways of selecting 4 face cards.
- (ii) In how many ways Ravi can select 4 cards from different suits?
- (iii) In how many ways Ravi can select all the 4 cards from same suit?

OR

Find the number of ways of selecting two cards red and two black.

- 38. Case-study 3:** Read the following passage and answer the questions given below.

Sanjay is father of Ram and grandfather of Ravi. Ravi is five years old. The age of Sanjay is sum of twice of Ram's age and thrice of Ravi's age. Let Sanjay's age is denoted by y and Ram's age is denoted by x .

- (i) Express the relation of Sanjay, Ram and Ravi's age as a linear function if age of Ram is 25 years, then what is the age of Sanjay?

- (ii) At what points the graph of the relation of Sanjay, Ram and Ravi's age intersects the x -axis and y -axis?

SOLUTIONS

- 1. (a) :** Number of ways of selecting a bus among 15 buses for going from A to $B = 15$

Since, the return journey is made on the same bus, therefore number of ways of selecting a bus for the return journey = 1

$\therefore$ Required number of ways = $15 \times 1 = 15$.

- 2. (b) :** Factors of a prime number are 1 and itself.

$\therefore A = \{1, p\}$

- 3. (b) :** $\tan(A + B) = \tan \frac{5\pi}{4} = \tan \left(\pi + \frac{\pi}{4} \right) = \tan \frac{\pi}{4}$

$\therefore \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$

$\Rightarrow (1 + \tan A) + \tan B(1 + \tan A) = 1 + 1$

(by adding 1 on both sides)

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

- 4. (a) :** $\frac{z_1}{z_2} = (2 + 3i) \left(\frac{1}{1 + 2i} \right) = \frac{(2 + 3i)(1 - 2i)}{(1 + 2i)(1 - 2i)}$

$$= (2 + 3i) \left(\frac{1}{5} - \frac{2}{5}i \right) = \left(\frac{2}{5} + \frac{6}{5} \right) + i \left(-\frac{4}{5} + \frac{3}{5} \right) = \frac{8}{5} - \frac{1}{5}i$$

- 5. (d) :** We have, $(3a + 5, 5 - b) = (8, -2)$

$\therefore 3a + 5 = 8$ and $5 - b = -2$

$\Rightarrow 3a = 3 \Rightarrow a = 1$ and $-b = -7 \Rightarrow b = 7$

- 6. (d) :** We have $\frac{5 - 2x}{3} \leq \frac{x}{6} - 5$ or $2(5 - 2x) \leq x - 30$

or $10 - 4x \leq x - 30$ or $-5x \leq -40$ or $x \geq 8$

Thus, all real numbers which are greater than or equal to 8 are the solutions of the given inequality i.e., $x \in [8, \infty)$.

- 7. (d) :** The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

E = The event "die shows 4" = $\{4\}$

F = The event "die shows an even number" = $\{2, 4, 6\}$

$\therefore E \cap F = \{4\} \neq \phi$

$\therefore E$ and F are not mutually exclusive.

Also, $E \cup F \neq S$

$\therefore E$ and F are not exhaustive

- 8. (a) :** Here, ${}^nC_2 - {}^nC_1 = 35$

$$\Rightarrow \frac{n(n-1)}{2} - n = 35 \Rightarrow n^2 - n - 2n = 70$$

$\Rightarrow n = 10$ or $n = -7$

$\Rightarrow n = 10$.

($\because n$ cannot be negative)

- 9. (c)**

10. (d) : We have $\lim_{x \rightarrow 2} \left(\frac{x^2 - 1}{x^4 - 2x^3 + 4x - 5} \right)$

$$= \frac{(2)^2 - 1}{(2)^4 - 2(2)^3 + 4(2) - 5} = \frac{4 - 1}{16 - 16 + 8 - 5} = \frac{3}{-3} = -1$$

11. (a) : Given, $\cos \theta = 3/5$ and θ lies in fourth quadrant.

Now, $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta} = \pm \sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm \frac{4}{5}$$

$$\Rightarrow \sin \theta = -\frac{4}{5}$$

[Since, $\sin \theta$ is negative in fourth quadrant]

Now, $\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{-5}{4}$.

12. (b) : Let us write the given equation in the form $(x^2 - 2x) + (y^2 + 4y) = 8$

Now, completing the squares, we get

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 1 + 4$$

or $(x - 1)^2 + (y + 2)^2 = 13$

Comparing it with the standard equation of the circle, $(x - h)^2 + (y - k)^2 = r^2$, we see that the centre of the circle is $(1, -2)$ and radius is $\sqrt{13}$.

13. (a) : We have, $a_n = a_{n-1} + 2$, $n \geq 2$ and $a_1 = 1$

$$\therefore a_2 = a_1 + 2 = 1 + 2 = 3, a_3 = a_2 + 2 = 3 + 2 = 5,$$

$$a_4 = a_3 + 2 = 5 + 2 = 7, a_5 = a_4 + 2 = 7 + 2 = 9.$$

$\therefore$ The corresponding series is $1 + 3 + 5 + 7 + 9 + \dots$

14. (c)

15. (a) : We have, $f(x) = \sin^2 x$

$$\therefore \frac{d}{dx} [f(x)] = \frac{d}{dx} (\sin x \sin x) \quad (\text{Using product rule})$$

$$= 2 \sin x \cos x = \sin 2x$$

16. (c) : The mean ($\bar{x}$) of the given data is,

$$\bar{x} = \frac{1}{20} \sum_{i=1}^{20} x_i = \frac{200}{20} = 10$$

The values of the deviations from mean, i.e., $|x_i - \bar{x}|$ are

2, 7, 8, 7, 6, 1, 7, 9, 10, 5, 2, 7, 8, 7, 6, 1, 7, 9, 10, 5

$$\therefore \sum_{i=1}^{20} |x_i - \bar{x}| = 124 \text{ and M.D. } (\bar{x}) = \frac{124}{20} = 6.2$$

17. (b) : Given, $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (4, -2)$

Slope of line, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{4 - 3} = -1$

$$\Rightarrow \tan \theta = -1 \Rightarrow \tan \theta = -\tan 45^\circ$$

$$\Rightarrow \tan \theta = \tan (180^\circ - 45^\circ) \Rightarrow \tan \theta = \tan 135^\circ$$

$$\Rightarrow \theta = 135^\circ$$

18. (a)

19. (c) : Let $A = (1 + \sqrt{11}, 0, 0)$ and $B = (1, -2, 3)$

$$\therefore AB = \sqrt{(1 - 1 - \sqrt{11})^2 + (-2 - 0)^2 + (3 - 0)^2}$$

$$= \sqrt{11 + 4 + 9} = \sqrt{24} = 2\sqrt{6} \text{ units}$$

$\therefore$ Hence, A is true but R is false

20. (a) : We have, $h(x) = \frac{x + \cos x}{\tan x}$... (i)

Differentiating both sides of (i) w.r.t. 'x', we get

$$\frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{(\tan x)^2}$$

Hence, both A and R are true and R is the correct explanation of A.

21. Let $R(x, y)$ be the required point.

Then, $PR = 2 \cdot RQ$ (Given) i.e., $PR : RQ = 2 : 1$. Thus, R divides PQ internally or externally in the ratio 2 : 1.

$\therefore$ Coordinates of R are given by

$$x = \frac{2 \times -2 + 1 \times 3}{2 + 1} = -\frac{1}{3} \text{ and } y = \frac{2 \times 5 + 1 \times -4}{2 + 1} = 2$$

So, the coordinates of R are $(-1/3, 2)$

If R divides PQ externally in the ratio 2 : 1, then the coordinates of R are given by

$$x = \frac{(2 \times -2) - (1 \times 3)}{2 - 1} = -7 \text{ and } y = \frac{(2 \times 5) - (1 \times -4)}{2 - 1} = 14$$

Hence, the coordinates of R are $(-7, 14)$.

OR

Given points are $A(1, 4)$, $B(3, -2)$ and $C(4, -5)$

Now, Slope of AB = $\frac{-2 - 4}{3 - 1} = \frac{-6}{2} = -3$

Slope of BC = $\frac{-5 - (-2)}{4 - 3} = \frac{-3}{1} = -3$ $\left[\because \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \right]$

$\therefore$ Slope of AB = Slope of BC

Hence, points A, B, C are collinear.

22. Let $z = \frac{1 + 7i}{(2 - i)^2} = \frac{1 + 7i}{(4 + i^2 - 4i)} = \frac{1 + 7i}{3 - 4i}$

$$= \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i} = \frac{3 + 4i + 21i + 28i^2}{9 - 16i^2}$$

$$= \frac{3 - 28 + 25i}{25} = \frac{-25 + 25i}{25} = -1 + i$$

23. L.H.S. = $\tan 70^\circ = \tan (50^\circ + 20^\circ) = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ}$

$$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\begin{aligned}\Rightarrow \tan 70^\circ (1 - \tan 50^\circ \tan 20^\circ) &= \tan 50^\circ + \tan 20^\circ \\ \Rightarrow \tan 70^\circ - \tan 70^\circ \tan 50^\circ \tan 20^\circ &= \tan 50^\circ + \tan 20^\circ \\ \Rightarrow \tan 70^\circ &= \tan 70^\circ \tan 50^\circ \tan 20^\circ + \tan 50^\circ + \tan 20^\circ \\ &= \cot 20^\circ \tan 50^\circ \tan 20^\circ + \tan 50^\circ + \tan 20^\circ \\ &= \tan 50^\circ + \tan 50^\circ + \tan 20^\circ = 2 \tan 50^\circ + \tan 20^\circ \\ &= \text{R.H.S}\end{aligned}$$

OR

$$\text{We have, } 2 \cos \theta = a + \frac{1}{a} \Rightarrow \cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right)$$

$$\text{Since, } \cos 2\theta = 2 \cos^2 \theta - 1$$

$$\begin{aligned}\therefore 2 \cos 2\theta &= 2(2 \cos^2 \theta - 1) = 2 \left[2 \left(\frac{1}{2} \left(a + \frac{1}{a} \right) \right)^2 - 1 \right] \\ &= 2 \left(\frac{2}{4} \left(a^2 + \frac{1}{a^2} + 2 \right) - 1 \right) = a^2 + \frac{1}{a^2} + 2 - 2 = \left(a^2 + \frac{1}{a^2} \right).\end{aligned}$$

24. The given inequations are $\frac{-2}{3} < 1 - \frac{x}{3} \leq \frac{2}{3}$

Taking the first inequations Taking the last inequations

$$1 - \frac{x}{3} > \frac{-2}{3}$$

$$\Rightarrow \frac{x}{3} < 1 + \frac{2}{3}$$

$$\Rightarrow x < 5$$

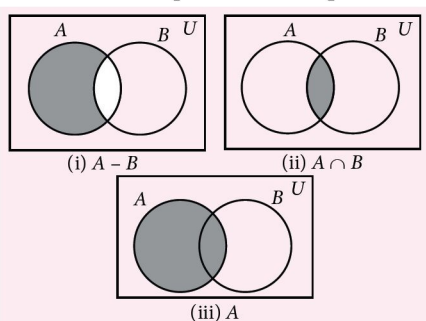
$$1 - \frac{x}{3} \leq \frac{2}{3}$$

$$\Rightarrow \frac{x}{3} \geq 1 - \frac{2}{3}$$

$$\Rightarrow x \geq 1$$

$$\therefore \text{Required solution set} = \{x : x < 5, x \in R\} \cap \{x : x \geq 1, x \in R\} = \{x : 1 \leq x < 5, x \in R\}.$$

25. The shaded area represents the required sets.



From Venn diagram (i), (ii) and (iii), it is clear that $(A - B) \cup (A \cap B) = A$

26. We have, $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5 : 3$

$$\begin{aligned}\Rightarrow \frac{\frac{n!}{3!(n-3)!}}{\frac{n!}{5!(n-5)!}} &= \frac{5}{3} \Rightarrow \frac{n!}{3!(n-3)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3} \\ \Rightarrow \frac{5 \times 4 \times 3! \times (n-5)!}{3!(n-3)(n-4)(n-5)!} &= \frac{5}{3} \Rightarrow \frac{20}{(n-3)(n-4)} = \frac{5}{3}\end{aligned}$$

$$\Rightarrow (n-3)(n-4) = 12 \Rightarrow n^2 - 7n + 12 = 12$$

$$\Rightarrow n^2 - 7n = 0 \Rightarrow n(n-7) = 0$$

$$\Rightarrow n = 0 \text{ or } n = 7$$

But $n \neq 0$, therefore $n = 7$.

27. Clearly, $(2x-1)^4 + 4(2x-1)^3(3-2x) + 6(2x-1)^2(3-2x)^2 + 4(2x-1)(3-2x)^3 + (3-2x)^4$

$$\begin{aligned}&= {}^4C_0(2x-1)^{4-0} + {}^4C_1(2x-1)^{4-1}(3-2x)^1 + \\ &{}^4C_2(2x-1)^{4-2}(3-2x)^2 + {}^4C_3(2x-1)^{4-3}(3-2x)^3 \\ &\quad + {}^4C_4(2x-1)^{4-4}(3-2x)^4 \\ &= [(2x-1) + (3-2x)]^4 = 2^4 = 16.\end{aligned}$$

OR

$$\begin{aligned}(1.02)^6 &= (1 + 0.02)^6 \\ &= {}^6C_0 + {}^6C_1(0.02)^1 + {}^6C_2(0.02)^2 + {}^6C_3(0.02)^3 + \\ &\quad {}^6C_4(0.02)^4 + {}^6C_5(0.02)^5 + {}^6C_6(0.02)^6 \\ &= 1 + 0.12 + 0.006 + 0.00016 + 0.0000024 + \dots \\ &= 1 + 0.126 + 0.00016\end{aligned}$$

(Ignoring other terms, as answer is required to correct upto four places of decimals)

$$= 1.12616.$$

SAMURAI SUDOKU

ANSWER - JANUARY 2023



3	8	6	1	9	2	5	7	4				1	6	2	3	9	4	8	7	5
9	2	5	7	6	4	8	1	3				8	5	9	1	2	7	6	4	3
1	7	4	8	3	5	2	6	9				4	3	7	8	6	5	2	9	1
2	1	9	4	7	8	3	5	6				6	4	8	9	5	2	3	1	7
6	3	8	9	5	1	4	2	7				3	9	5	6	7	1	4	8	2
4	5	7	6	2	3	9	8	1				2	7	1	4	3	8	9	5	6
7	4	1	2	8	9	6	3	5	9	1	2	7	8	4	2	1	3	5	6	9
5	6	2	3	4	7	1	9	8	3	7	4	5	2	6	7	4	9	1	3	8
8	9	3	5	1	6	7	4	2	5	8	6	9	1	3	5	8	6	7	2	4
									2	6	4	8	3	7	1	5	9			
									5	8	3	6	9	1	4	7	2			
									9	7	1	2	4	5	6	3	8			
4	6	3	5	2	7	8	1	9	7	6	3	2	4	5	9	3	7	6	8	1
1	5	9	8	6	4	3	2	7	4	5	6	8	6	1	4	2	5	3	9	7
2	8	7	9	1	3	4	5	6	1	2	8	3	9	7	8	6	1	5	2	4
3	2	5	6	4	9	7	8	1				5	2	4	6	1	9	7	3	8
7	9	8	3	5	1	2	6	4				1	8	9	3	7	2	4	6	5
6	1	4	2	7	8	9	3	5				7	3	6	5	4	8	9	1	2
9	4	6	1	8	2	5	7	3				4	5	8	1	9	3	2	7	6
8	3	1	7	9	5	6	4	2				9	1	2	7	5	6	8	4	3
5	7	2	4	3	6	1	9	8				6	7	3	2	8	4	1	5	9

28. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let e be its eccentricity.

It is given that the coordinates of foci are $(\pm 5, 0)$ and the equation of one of the directrices is $x = 36/5$.

$$\therefore ae = 5 \text{ and } \frac{a}{e} = \frac{36}{5} \Rightarrow ae \times \frac{a}{e} = 5 \times \frac{36}{5}$$

$$\Rightarrow a^2 = 36 \Rightarrow a = 6$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = a^2 - (ae)^2 = 36 - 25 = 11 \Rightarrow b = \sqrt{11}$$

Substituting $a = 6$ and $b = \sqrt{11}$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we

obtain $\frac{x^2}{36} + \frac{y^2}{11} = 1$ as the required equation of ellipse.

OR

Given, $e = 2/3$ and length of latus rectum = 5

$$\Rightarrow \frac{2b^2}{a} = 5 \Rightarrow b^2 = \frac{5a}{2} \quad \dots (i)$$

$$\Rightarrow \frac{5a}{2} = a^2 \left(1 - \frac{4}{9}\right) \Rightarrow \frac{5}{2} = \frac{5a}{9} \Rightarrow a = \frac{9}{2} \Rightarrow a^2 = \frac{81}{4}$$

$$\text{and } b^2 = \frac{5 \times 9}{2 \times 2} = \frac{45}{4} \quad (\text{From (i)})$$

Hence, the required equation of the ellipse is

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1 \text{ or } \frac{4x^2}{45} + \frac{4y^2}{81} = 1$$

$$\mathbf{29.} \text{ We have } \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} - \frac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha}}{1 + \frac{\sin \alpha}{\cos \alpha} \times \frac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha}} \quad \left[\because \tan \beta = \frac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha} \right]$$

$$= \frac{\sin \alpha - n \sin^3 \alpha - n \sin \alpha \cos^2 \alpha}{\cos \alpha (1 - n \sin^2 \alpha) + n \sin^2 \alpha \cos \alpha}$$

$$= \frac{\sin \alpha - n \sin^3 \alpha - n \sin \alpha (1 - \sin^2 \alpha)}{\cos \alpha - n \sin^2 \alpha \cos \alpha + n \sin^2 \alpha \cos \alpha}$$

$$= \frac{\sin \alpha - n \sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\cos \alpha} (1 - n) = (1 - n) \tan \alpha$$

OR

Refer to answer 20, Page no. 92, Class-11, MTG 100 percent Mathematics.

30. Total number of possible outcomes of throwing three dice simultaneously = $6 \times 6 \times 6 = 6^3 = 216$

(i) All dice can show the same face in one of the following mutually exclusive ways:

$(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)$

So, number of favourable outcomes = 6

$$\therefore \text{Required probability} = \frac{6}{6^3} = \frac{1}{36}$$

(ii) The total number of ways in which all dice show different faces is equal to the number of ways of arranging 6 distinct objects by taking three at a time i.e., ${}^6C_3 \times 3!$.

So, number of favourable outcomes = ${}^6C_3 \times 3!$

$$\therefore \text{Required probability} = \frac{{}^6C_3 \times 3!}{6^3} = \frac{5}{9}$$

31. Here, $A = \{2, 3\}$, $B = \{6, 8\}$, $C = \{1, 2\}$ and $D = \{6, 9\}$
Now, $A \times B = \{2, 3\} \times \{6, 8\} = \{(2, 6), (2, 8), (3, 6), (3, 8)\}$
 $C \times D = \{1, 2\} \times \{6, 9\} = \{(1, 6), (1, 9), (2, 6), (2, 9)\}$

$$\therefore (A \times B) \cap (C \times D) = \{(2, 6)\} \quad \dots (i)$$

Also, $A \cap C = \{2\}$ and $B \cap D = \{6\}$

$$\therefore (A \cap C) \times (B \cap D) = \{2\} \times \{6\} = \{(2, 6)\} \quad \dots (ii)$$

Thus, from (i) and (ii) we can say that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

32. (i) We first show that

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$$

Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$$

$$\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C)$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

$$\text{Thus, } A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \quad \dots (1)$$

Now, we will show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

Let $x \in (A \cup B) \cap (A \cup C) \Rightarrow x \in A \cup B$ and $x \in A \cup C$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C) \Rightarrow x \in A \text{ or } (x \in B \cap C)$$

$$\Rightarrow x \in A \cup (B \cap C)$$

$$\text{Thus, } (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \quad \dots (2)$$

So, from (1) and (2), we have

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(ii) Let $x \in A \cap (B \cup C)$ then $x \in A$ and $x \in (B \cup C)$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

$$\text{Thus, } A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad \dots (1)$$

Again if $x \in (A \cap B) \cup (A \cap C)$

then $x \in (A \cap B)$ or $x \in (A \cap C)$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow x \in A \text{ and } x \in (B \cup C) \Rightarrow x \in A \cap (B \cup C)$$

$$\text{Thus, } (A \cap B) \cup (A \cap C) \subseteq (A \cap (B \cup C)) \quad \dots (2)$$

From (1) and (2), we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

33. We compute the following table :

Class interval	Mid points (x_i)	f_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$	u_i^2	$f_i u_i^2$
45-55	50	7	-4	-28	16	112
55-65	60	20	-3	-60	9	180
65-75	70	27	-2	-54	4	108
75-85	80	23	-1	-23	1	23
85-95	90	13	0	0	0	0
95-105	100	6	1	6	1	6
105-115	110	3	2	6	4	12
115-125	120	1	3	3	9	9
Total		100		-150		450

Let the assumed mean, $A = 90$

$$\text{Now, } \bar{x} = A + \frac{\sum f_i u_i}{N} \times h = 90 + \left(\frac{-150}{100} \right) \times 10$$

$$= 90 - 15 = 75$$

$$\begin{aligned} \text{Variance, } \sigma^2 &= h^2 \left[\frac{1}{N} \sum f_i u_i^2 - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right] \\ &= 100 \times \left[\frac{1}{100} \times 450 - \left(\frac{1}{100} (-150) \right)^2 \right] = 100 \times [4.5 - 2.25] \\ &= 100 \times (2.25) = 225 \end{aligned}$$

$$\text{So, standard deviation, } \sigma = \sqrt{225} = 15$$

OR

Refer to answer 19, Page no. 414, Class-11, MTG 100 percent Mathematics.

34. Let a be the first term and r be the common ratio of the G.P. Then,

$$S_1 = a \left(\frac{r^n - 1}{r - 1} \right), S_2 = a \left(\frac{r^{2n} - 1}{r - 1} \right) \text{ and } S_3 = a \left(\frac{r^{3n} - 1}{r - 1} \right).$$

$$\begin{aligned} \text{Now, } S_1(S_3 - S_2) &= a \left(\frac{r^n - 1}{r - 1} \right) \left\{ a \left(\frac{r^{3n} - 1}{r - 1} \right) - a \left(\frac{r^{2n} - 1}{r - 1} \right) \right\} \\ &= \frac{a^2}{(r - 1)^2} (r^n - 1) \{ (r^{3n} - 1) - (r^{2n} - 1) \} \\ &= \frac{a^2}{(r - 1)^2} (r^n - 1) (r^{3n} - r^{2n}) = \frac{a^2}{(r - 1)^2} (r^n - 1) r^{2n} (r^n - 1) \\ &= \left\{ ar^n \left(\frac{r^n - 1}{r - 1} \right) \right\}^2 \text{ and} \end{aligned}$$

$$\text{Also, } (S_2 - S_1)^2 = \left\{ a \left(\frac{r^{2n} - 1}{r - 1} \right) - a \left(\frac{r^n - 1}{r - 1} \right) \right\}^2$$

$$\begin{aligned} &= \frac{a^2}{(r - 1)^2} \{ (r^{2n} - 1) - (r^n - 1) \}^2 \\ &= \frac{a^2}{(r - 1)^2} \{ r^n (r^n - 1) \}^2 = \left\{ ar^n \left(\frac{r^n - 1}{r - 1} \right) \right\}^2 \end{aligned}$$

$$\text{Hence, } S_1(S_3 - S_2) = (S_2 - S_1)^2$$

35. Let $f(x) = \cos(x^2 + 1)$

$$\therefore f(x + h) = \cos((x + h)^2 + 1)$$

$$\text{Now, using first principle, } f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\cos\{(x + h)^2 + 1\} - \cos(x^2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{(x + h)^2 + 1 + x^2 + 1}{2}\right) \sin\left(\frac{(x + h)^2 + 1 - x^2 - 1}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(x^2 + 1 + xh + \frac{h^2}{2}\right) \sin\left(xh + \frac{h^2}{2}\right)}{h} \\ &= -2 \lim_{h \rightarrow 0} \sin\left(x^2 + 1 + xh + \frac{h^2}{2}\right) \lim_{h \rightarrow 0} \frac{\sin\left(xh + \frac{h^2}{2}\right)}{h} \end{aligned}$$

$$= -2 \sin(x^2 + 1) \lim_{h \rightarrow 0} \frac{\sin h \left(x + \frac{h}{2} \right)}{h \left(x + \frac{h}{2} \right)} \times \left(x + \frac{h}{2} \right)$$

$$= -2 \sin(x^2 + 1) \times x = -2x \sin(x^2 + 1)$$

OR

Refer to answer 25, Page no. 370, Class-11, MTG 100 percent Mathematics.

36. (i) : Hina's house is situated at $A(1, 2, 3)$ and her friend's home is located at $B(3, 3, 5)$. Distance between Hina's house and her friend's home is

$$\begin{aligned} AB &= \sqrt{(3-1)^2 + (3-2)^2 + (5-3)^2} \\ &= \sqrt{(2)^2 + (1)^2 + (2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3 \text{ units} \end{aligned}$$



Recipe for Success

“Never let success get to your head and never let failure get to your heart”

— Drake

(ii) First, Hina went to her friend's home and then to school.

$$\begin{aligned}\therefore \text{Total distance covered by her is } AB + BC \\ = \sqrt{2^2 + 1^2 + 2^2} + \sqrt{0 + 3^2 + 0} = \sqrt{4 + 1 + 4} + \sqrt{9} \\ = 3 + 3 = 6 \text{ units}\end{aligned}$$

(iii) The shortest distance between Hina's home to her school is $AC = \sqrt{(3-1)^2 + (6-2)^2 + (5-3)^2} = \sqrt{24} = 2\sqrt{6}$ units

OR

Let the coordinate of park i.e., $D(\alpha, \beta, \gamma)$ is equidistant from $A(1, 2, 3)$ and $B(3, 3, 5)$

$$\text{Then, } AD^2 = BD^2$$

$$\begin{aligned}\Rightarrow (\alpha - 1)^2 + (\beta - 2)^2 + (\gamma - 3)^2 \\ = (\alpha - 3)^2 + (\beta - 3)^2 + (\gamma - 5)^2 \\ \Rightarrow \alpha^2 + 1 - 2\alpha + \beta^2 + 4 - 4\beta + \gamma^2 + 9 - 6\gamma \\ = \alpha^2 + 9 - 6\alpha + \beta^2 + 9 - 6\beta + \gamma^2 + 25 - 10\gamma \\ \Rightarrow 4\alpha + 2\beta + 4\gamma - 29 = 0\end{aligned}$$

So, the locus of park is $4x + 2y + 4z - 29 = 0$.

37. (i) There are 12 face cards out of which 4 cards are selected.

$$\therefore \text{Required number of ways} = {}^{12}C_4 = \frac{12!}{4!8!} = 495$$

(ii) One card must be drawn from each suit.

$$\therefore \text{Required number of ways} \\ = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{12}C_1 = (13)^3 \times 12 = 26364.$$

(iii) There are 3 suits of 13 cards each and 1 suit of 12 cards.

$\therefore$ There are ${}^{13}C_4$ ways of choosing 4 diamond cards, ${}^{13}C_4$ ways of choosing 4 club cards, ${}^{13}C_4$ ways of choosing 4 heart cards and ${}^{12}C_4$ ways of choosing 4 spade cards

$$\begin{aligned}\therefore \text{Required number of ways} &= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{12}C_4 \\ &= (3 \times {}^{13}C_4) + {}^{12}C_4 \\ &= \frac{3 \times 13!}{4!9!} + \frac{12!}{4!8!} = 2145 + 495 = 2640.\end{aligned}$$

OR

There are 26 red cards and 25 black cards, 2 red and 2 black cards need to be chosen.

$$\begin{aligned}\therefore \text{Required number of ways} &= {}^{26}C_2 \times {}^{25}C_2 \\ &= \frac{26 \times 25}{2 \times 1} \times \frac{25 \times 24}{2 \times 1} = 97500\end{aligned}$$

38. (i) Let x be the age of Ram, then the relation of Sanjay, Ram and Ravi's age is $y = 2x + 15$, which is a linear function.

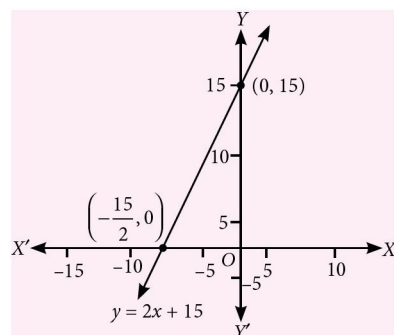
Putting $x = 25$ in $y = 2x + 15$, we get $y = 2 \times 25 + 15$

$$\Rightarrow y = 65 \text{ years}$$

$\therefore$ Age of Sanjay = 65 years

(ii) We have, $y = 2x + 15$ (given)

Let us draw the graph of given function.



The graph intersects the X-axis and Y-axis at

$\left(-\frac{15}{2}, 0\right)$ and $(0, 15)$ respectively.



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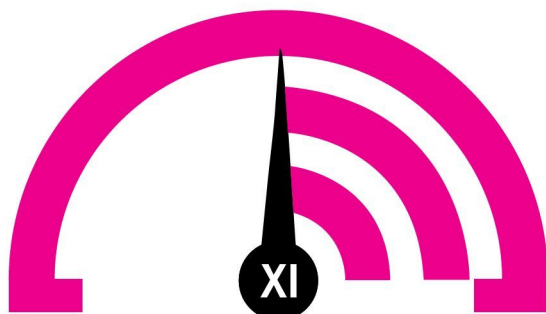
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MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 7 : Limits and Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

1. $\lim_{x \rightarrow 0} \frac{(\cos x)^{1/3} - (\cos x)^{1/2}}{\sin^2 x}$ equals
(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
2. The value of $\lim_{x \rightarrow \infty} \frac{e^{1/x^2} - 1}{2 \tan^{-1} x^2 - \pi}$ is equal to
(a) 1 (b) 2 (c) $-1/2$ (d) $1/4$
3. If $f(x) = \frac{5x}{(1-x)^{2/3}} + \cos^2(2x+1)$, then $f'(0) =$
(a) $5 + 2 \sin 2$ (b) $5 + 2 \cos 2$
(c) $5 - 2 \sin 2$ (d) $5 - 2 \cos 2$
4. If $f'(4) = 5$, $g'(4) = 12$, $f(4)g(4) = 2$ and $g(4) = 6$, then $\left(\frac{f}{g}\right)'(4) =$
(a) $\frac{5}{36}$ (b) $\frac{11}{18}$ (c) $\frac{23}{36}$ (d) $\frac{13}{18}$
5. If f be a function such that $f(9) = 9$ and $f'(9) = 3$, then $\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to
(a) 9 (b) 3
(c) 1 (d) None of these
6. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{\cos^{-1} \left[\frac{1}{4} (3 \sin x - \sin 3x) \right]}$, where $[\cdot]$ denotes greatest integer function, is
(a) $2/\pi$ (b) 1
(c) $4/\pi$ (d) Does not exist

One or More Than One Option(s) Correct Type

7. Let $f(x) = x + \sqrt{x^2 + 2x}$ and $g(x) = \sqrt{x^2 + 2x} - x$, then
(a) $\lim_{x \rightarrow \infty} g(x) = 1$ (b) $\lim_{x \rightarrow \infty} f(x) = 1$
(c) $\lim_{x \rightarrow -\infty} f(x) = -1$ (d) $\lim_{x \rightarrow -\infty} g(x) = -1$
8. The function $y = \sin^{-1}(\cos x)$ is not differentiable at
(a) $x = \pi$ (b) $x = -2\pi$
(c) $x = 2\pi$ (d) $x = \pi/2$
9. If $\lim_{x \rightarrow 0} \frac{k + \cos lx}{x^2}$ exists and has the value equal to -4 , then the ordered pair(s) (k, l) is
(a) $(-1, 2\sqrt{2})$ (b) $(-1, 2)$
(c) $(-1, -2\sqrt{2})$ (d) $(-1, -2)$
10. Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = x^2 + xf'(2) + f''(3)$, then
(a) $f'(1) = 4 + f'(2)$
(b) $g'(2) = 8 + g'(1)$
(c) $g''(2) + f''(3) = 4$
(d) None of these
11. Let $f(x) = (ax + b) \cos x + (cx + d) \sin x$ and $f'(x) = x \cos x$ be an identity in x , then
(a) $a = 0$ (b) $b = 1$ (c) $c = 1$ (d) $d = 0$
12. Let $f(x) = 1 + e^{\ln(\ln x)} \cdot \ln(k^2 + 25)$ and $g(x) = \frac{1}{|x| - 1}$. If $\lim_{x \rightarrow 1} (f(x))^{g(x)} = k(2 \sin^2 \alpha + 3 \cos \beta + 5)$, $k > 0$ and $\alpha, \beta \in R$, then which of the following is (are) correct?

- (a) The value of k is equal to 5.
 (b) The value of $\frac{\sin^{10}\alpha + \cos^5\beta}{\sin^2\alpha + \cos\beta}$ is equal to 1.
 (c) $\cos^2\beta + \sin^4\alpha = 2$
 (d) $\sin^2\alpha > \cos\beta$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let $f(x)$ be a cubic polynomial and $g(x)$ be a quadratic polynomial with same leading coefficients. Also, let $g(x) = 0$ has roots α and β . If $\lim_{x \rightarrow \alpha} \frac{f(x)}{g(x)} = 1$ and $\lim_{x \rightarrow \beta} \frac{f(x)}{g(x)} = 4$, then

13. The value of $\beta - \alpha$ is
 (a) 1 (b) 2 (c) 3 (d) 4
14. $\lim_{x \rightarrow \alpha} (1 + g(x))^{\frac{1}{f(x)}} =$
 (a) 1 (b) e (c) e^2 (d) $1/e$
15. If $\lim_{x \rightarrow \beta} (1 + f(x))^{1/g(x)} = e^n$, then value of n is
 (a) 1 (b) 0 (c) 2 (d) 4

Matrix Match Type

16. Match the following :

Column-I		Column-II	
(P)	$\lim_{x \rightarrow 2^+} \{x\} \frac{\sin(x-2)}{(x-2)^2} =$	(1)	-3
(Q)	$\lim_{x \rightarrow 2} [2-x] + [x-2] - x =$	(2)	1
(R)	$\lim_{x \rightarrow 2} \{2-x\} + \{x-2\} =$	(3)	-1
(S)	$\lim_{x \rightarrow 0^-} \frac{\frac{1}{e^x} - \frac{-1}{e^x}}{\frac{1}{e^x} + \frac{-1}{e^x}} =$	(4)	2

- | | | | | |
|-----|---|---|---|---|
| | P | Q | R | S |
| (a) | 4 | 2 | 3 | 1 |
| (b) | 2 | 1 | 2 | 3 |
| (c) | 4 | 1 | 3 | 2 |
| (d) | 1 | 4 | 2 | 3 |

Numerical Answer Type

17. Let $\lim_{x \rightarrow 1} \frac{x^6 + 1 + \frac{5}{3}(x^4 + x^2) - \frac{8}{3}(x^5 + x)}{x^5 + x + 6x^3 - 4x^4 - 4x^2} = \frac{p}{q}$, where $p, q \in \mathbb{N}$. The smallest value of $(p + q)$ is _____.
18. Let $F(x) = f(x) g(x) h(x)$ for all real x , where $f(x)$, $g(x)$ and $h(x)$ are differentiable functions. At some point x_0 , $F'(x_0) = 21 F(x_0)$, $f'(x_0) = 4f(x_0)$, $g'(x_0) = -7g(x_0)$ and $h'(x_0) = kh(x_0)$. Then, $k/4$ is equal to _____.
19. If $f(x) = \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}$, then $\lim_{x \rightarrow \infty} f(x)$ is equal to _____.
20. If $y = \frac{a + bx}{c + dx}$, where a, b, c, d are constants and $\lambda y_1 y_3 = \mu y_2^2$, then the minimum integral value of $\mu \lambda^2$ must be equal to _____.



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9

Study Tips to Ace Your CBSE Class 12 Maths Exam With Full Marks!

CBSE class 12 examinations are just around the corner and there's much speculation going on about last-minute preparation tips, particularly for subjects like Mathematics as the exam pattern of these papers is devised in such a manner so as to test a student's mettle in terms of analytical thinking and reasoning ability. Maths comprises concepts that are understood through regular practice. As a core subject, Mathematics requires a thorough knowledge of theories and correct application of formulae through stepwise logical approaches.

Here are some tips to help students come out of 'Maths phobia', prepare well for the exam and excel in the subject.

1 Understand the Contents and Weightage of the Syllabus

Firstly, go through the latest CBSE Class 12th Maths syllabus 2022-23, where you will find the weightage of different units.

- Differential Equations consisting of the application based difficult questions.
- Calculus, which carries a whopping 44% weightage, and can be really scoring.
- Vectors and 3D geometry carry the next highest 18% weightage.
- Probability, Relations & Functions and Algebra have a weightage of 10%, 10% and 13% respectively.
- Linear Programming carry 6% weightage.

2 Prepare NCERT Textbooks

NCERT textbooks offer an exhaustive range of exercise, questions and solved examples on every topic, which is more than enough to do well in your board examinations.

3 Practice the Mathematical Concepts by Writing them Down

When doing exercises or practicing questions, using notes as a guide and writing the procedures used to complete the exercise. It is not necessary to write the procedure if it is familiar to you and on tips. Write down what will help you when solving a Math problem. Making notes on important points while studying will help you remember them later when you revise. Maintain a separate register for formulae, theories and methods.

4 Regular in Solving Maths Questions

Consistency is the key to success in any endeavour, not just math. Math is a practice subject; it is simple to train and master the brain, repeated practice will aid individuals in remembering it in the long run.

5 Try Out the Most Difficult Math Topics

Working on topics with friends can help you understand any subject where you have an opinion. It will also provide opportunities for discussion of concepts and skills. For those who struggle with numbers, group study can alleviate anxiety and trouble. Working with friends on the most difficult topics can result in higher grades, better understanding and more systematic problem-solving. Individuals can also hire personal tutors or study at institutional centres.

6 Find Solution Yourself

It is good to go through various types of problems, but at the same time, you should also make sure that you solve them for yourself. It is easy to learn the theories and concepts but to learn their application is not that simple. So, if you want to score full marks in mathematics, you need to solve each question by yourself, at least 3 to 4 times.

7 Practice Sample Papers and Previous Years' Papers

Examiners follow a pattern of repetition for specific questions over a number of years, that is why it is very important to go through previous years' papers.

Make sure that you solve the last few year's papers on a timer, it will improve your speed, reasoning ability and time management skills.

8 Always Practice Maths in a Calm Environment

Working in a calm environment allows for better understanding. If your home is frequently too noisy or has too many distractions, go to the library or another quiet place to study. A library, media library, or a bedroom stand can all help with this. Working in a calm environment allows you to better understand and concentrate in one location.

9 Presentation is Very Important

While writing the exam neat work is a non-negotiable perk that always benefits a student. Make sure to label the graphs and figures properly. Avoid overwriting and scribbling, if you've made a mistake, then strike it off and start from below. Always leave some space between two questions, and start a new section on a fresh page.

In short, if you're a little rusty in Math, these easy tips may help you overcome all of the obstacles by brushing up on your Mathematical brain. All of these techniques combine to form a good blend of practicality, logic, and consistency, which will help anyone perform better in Math. Not just for those who are afraid of math, but also for those who want to improve and excel in the subject.

We wish you all the best in your board exam preparation!

For more such articles, visit blog.mtg.in





CBSE warm-up!

Exam on
11th March
2023

CLASS-XII

Practice questions for CBSE Exams as per the latest pattern
and reduced syllabus by CBSE for the academic session 2022-23.

Practice Paper 2022-23

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

- The elements a_{ij} of a 3×3 matrix are given by $a_{ij} = \frac{1}{2}|-3i + j|$. Write the value of element a_{32} .
(a) $3/2$ (b) $7/2$ (c) $5/2$ (d) $9/2$
- Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/4$
- If $y^2(2-x) = x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to
(a) 2 (b) -2 (c) 3 (d) $-3/2$
- If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is
(a) 3 (b) 0 (c) 9 (d) 27
- If the inverse of the matrix $\begin{bmatrix} \alpha & 7 & -1 \\ 2 & 2 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ does not exist, then the value of α is
(a) $29/6$ (b) $31/6$ (c) $1/7$ (d) $33/6$

- If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is

- $\begin{bmatrix} 1 & 2^n - 2 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & n^2 \\ 1 & 1 \end{bmatrix}$

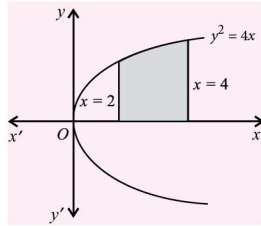
- Find : $\int \sin^{-1}(2x) dx$

- $x \cos^{-1}(2x) - \frac{1}{2} \sqrt{1-4x^2} + C$
- $x \cos^{-1}(2x) + \frac{1}{2} \sqrt{1-4x^2} + C$
- $x \sin^{-1}(2x) + \frac{1}{2} \sqrt{1-4x^2} + C$
- $x \sin^{-1}(2x) - \frac{1}{2} \sqrt{1-4x^2} + C$

- Solve : $\cos x (1 + \cos y) dx - \sin y (1 + \sin x) dy = 0$
(a) $(1 - \sin x)(1 + \cos y) = C$
(b) $(1 + \sin x)(1 + \cos y) = C$
(c) $(1 + \sin x)(1 - \cos y) = C$
(d) $(1 - \sin x)(1 - \cos y) = C$

9. Find the area of shaded region of the given graph.

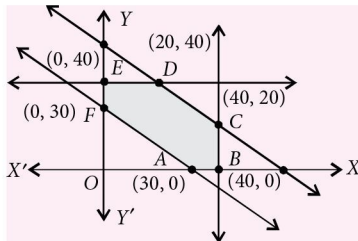
- (a) $\frac{4}{3}(8+2\sqrt{2})$ sq. units
 (b) $\frac{4}{3}(8-2\sqrt{2})$ sq. units
 (c) $\frac{3}{4}(8-2\sqrt{2})$ sq. units
 (d) $\frac{3}{4}(8+2\sqrt{2})$ sq. units



10. If $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, then R^5 equals

- (a) R (b) $2R$ (c) $-R$ (d) $-2R$

11. Feasible region for a LPP is shown in the following figure.

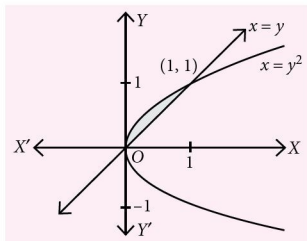


Minimum of $Z = 3x + 4y + 270$ occurs at the point
 (a) (20, 40) (b) (30, 0) (c) (0, 40) (d) (0, 30)

12. If $P(A) = \frac{6}{10}$, $P(B) = \frac{2}{7}$ and $P(A \cup B) = \frac{3}{5}$, then find the value of $P(B|A)$.
 (a) $4/7$ (b) $15/21$ (c) $1/7$ (d) $10/21$

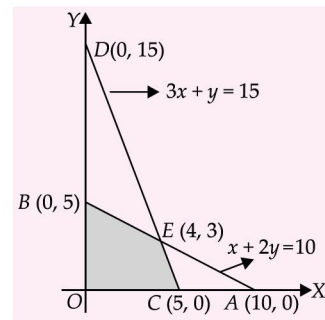
13. The function $(x - \sin x)$ decreases for
 (a) all x (b) $x < \pi/2$
 (c) $0 < x < \pi/4$ (d) No value of x

14. Using integration, find the area of the shaded region.
 (a) $1/2$ sq. unit
 (b) $1/3$ sq. unit
 (c) $1/5$ sq. unit
 (d) $1/6$ sq. unit



15. The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on
 (a) $(-1, \infty)$ (b) $(-\infty, 0)$
 (c) $(-\infty, \infty)$ (d) None of these

16. Maximise $Z = 3x + 2y$, if the shaded region is the feasible region.



- (a) Max. $Z = 10$ (b) Max. $Z = 15$
 (c) Max. $Z = 18$ (d) None of these

17. The value of $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ is

- (a) $\frac{\pi}{32}$ (b) $\frac{\pi^2}{32}$ (c) $\frac{\pi}{16}$ (d) $\frac{\pi^2}{16}$

18. The equation of the line in vector form passing through the point $(-1, 3, 5)$ and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}, z=2$ is

- (a) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$
 (b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$
 (c) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$
 (d) $\vec{r} = (2\hat{i} + 3\hat{j}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. Assertion (A): If $|\vec{a}| = 2, |\vec{b}| = 3, |2\vec{a} - \vec{b}| = 5$, then $\vec{a} \cdot \vec{b} = 0$.

Reason (R): If vectors $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}|$, then $\theta = 90^\circ$.

20. Assertion (A): The order of the differential equation of all conics whose centre lies at $(0, 0)$ is 2.
 Reason (R): The order of the differential equation whose solution is $y = C_1 e^{2x+C_2} + C_3 e^{2x+C_4}$ is 1.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the value of $\sin\left(\cot^{-1}\frac{3}{4}\right)$.

OR

Find the value of $\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right)$.

22. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 500$ and $|\vec{b}| = 5$, then find the value of $|\vec{a}|$.

23. Show that $f(x) = [x]$ is not differentiable at $x = 1$.

OR

Check whether the function

$$f(x) = \begin{cases} (3^{\sin x} - 1)^2, & \text{for } x \neq 0 \\ 2\log 3, & \text{for } x = 0 \end{cases}$$

is continuous or not.

24. The volume of a cube is increasing at the rate of $9 \text{ cm}^3/\text{s}$. How fast is its surface area increasing when the length of an edge is 10 cm ?
25. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Solve the following LPP graphically :
Minimize $z = 5x + 7y$
Subject to the constraints
 $2x + y \geq 8$; $x + 2y \geq 10$; $x, y \geq 0$

27. Evaluate : $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$

OR

Evaluate : $\int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$

28. Solve the differential equation: $y + x \frac{dy}{dx} = x - y \frac{dy}{dx}$

OR

Find the General solution of the differential equation $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$.

29. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', then what is the probability that she threw 3, 4, 5 or 6 with the die?

OR

A speaks truth in 60% of the cases, while B in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact?

30. Evaluate : $\int_0^{\pi/4} \frac{\sin 2\theta}{\sin^4 \theta + \cos^4 \theta} d\theta$

31. Find the distance between the lines given by $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Find the product BA of the given matrices

$$A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Also, use it in solving the equations : $x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$.

33. Show that the function $f : (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

OR

Given a function defined by $f(x) = \sqrt{4-x^2}$, $0 \leq x \leq 2$, $0 \leq f(x) \leq 2$. Show that f is bijective function.

34. Find the image of the point $(2, -1, 5)$ in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also, find the equation of the line joining the given point and its image. Find the length of that line segment also.

OR

Find the shortest distance and the vector equation of the line of shortest distance between the lines given by $\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

35. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

- 36. Case-Study 1 :** Read the following passage and answer the questions given below.

Varun and Isha decided to play with dice to keep themselves busy at home as their schools are closed due to coronavirus pandemic. Varun throw a dice repeatedly until a six is obtained. He denotes the number of throws required by X .



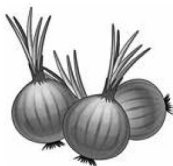
- Find the probability that $X = 2$.
- Find the probability that $X = 4$.
- Find the value of $P(X \geq 6)$.

OR

Find the value of $P(X \geq 4)$.

- 37. Case-Study 2 :** Read the following passage and answer the questions given below.

The Government declare that farmers can get ₹ 600 per quintal for their onions on 1st August and after that, the price will be dropped by ₹ 3 per quintal per extra day. Shyam's father has 80 quintal of onions in the field on 1st August and he estimates that crop is increasing at the rate of 1 quintal per day.



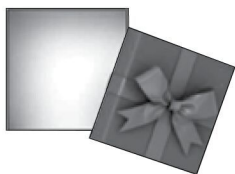
- If x is the number of days after 1st August, then represent the price and quantity of onion in terms of x .
- Represent revenue R as a function of x .
- Find the number of days after 1st August, when Shyam's father attain maximum revenue.

OR

Find the maximum revenue.

- 38. Case-Study 3 :** Read the following passage and answer the questions given below.

Megha wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square piece of cardboard of side 20 cm. Assume the height of box as x .



- Find the volume (V) of the open box formed by folding up the cutting corner.
- Find the value of x , if rate of change of volume with respect to height will be zero.

SOLUTIONS

1. (b) : The value of element $a_{32} = \frac{1}{2}|-3(3)+2| = \frac{7}{2}$

2. (d) : Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \theta \Rightarrow \sin \theta = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$

$$\Rightarrow \theta = \frac{\pi}{4} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$\therefore$ Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

3. (a) : Given, $y^2(2-x) = x^3$

$$\Rightarrow y^2 = \frac{x^3}{2-x} \Rightarrow \frac{dy}{dx} = \frac{6x^2 - 2x^3}{2y(2-x)^2} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2$$

4. (b) : We have, $A^T = -A$ [$\because A$ is skew-symmetric matrix]

$\therefore |A^T| = |-A| \Rightarrow |A| = (-1)^3|A|$ [$\because A$ is of order 3]

$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

5. (d) : Let $A = \begin{bmatrix} \alpha & 7 & -1 \\ 2 & 2 & 1 \\ 3 & 2 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} \alpha & 7 & -1 \\ 2 & 2 & 1 \\ 3 & 2 & 4 \end{vmatrix}$

$= \alpha(8-2) - 7(8-3) - 1(4-6) \Rightarrow |A| = 6\alpha - 33$

Now, as A^{-1} does not exist, therefore $|A| = 0$

$\Rightarrow 6\alpha - 33 = 0 \Rightarrow \alpha = 33/6$

6. (c) : $A^2 = \begin{bmatrix} 1 & 2 \times 2 \\ 0 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 3 \times 2 \\ 0 & 1 \end{bmatrix}$

Similarly, $A^4 = \begin{bmatrix} 1 & 4 \times 2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^n = A^{n-1}A = \begin{bmatrix} 1 & n \times 2 \\ 0 & 1 \end{bmatrix} \therefore A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

7. (c) : Let $I = \int \sin^{-1}(2x) dx = \int 1 \cdot \sin^{-1}(2x) dx$

Integrating by parts, we get

$$= \sin^{-1}(2x)x - \int \left(\frac{2}{\sqrt{1-4x^2}}\right) x dx = x \sin^{-1}(2x) + \int \frac{dt}{4\sqrt{t}}$$

(On putting $1 - 4x^2 = t \Rightarrow -8x dx = dt$)

$$= x \sin^{-1}(2x) + \frac{2}{4}(t)^{1/2} + C = x \sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$$

8. (b) : The given equation is

$\cos x (1 + \cos y) dx - \sin y (1 + \sin x) dy = 0$

$$\Rightarrow \frac{\cos x}{1 + \sin x} dx - \frac{\sin y}{1 + \cos y} dy = 0$$

After integrating we get

$(1 + \sin x) (1 + \cos y) = C$

9. (b) : Required area = $\int_2^4 y dx$
 $= \frac{4}{3}(8 - 2\sqrt{2})$ sq. units

10. (a) : $R^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = R$

$\therefore R^4 = R^2 = R \Rightarrow R^5 = R^4 \cdot R = R \cdot R = R^2 = R$

11. (b)

12. (d) : Given, $P(A) = \frac{6}{10}, P(B) = \frac{2}{7}, P(A \cup B) = \frac{3}{5}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{7}$

Now, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/7}{6/10} = \frac{10}{21}$

13. (d) : Let $f(x) = x - \sin x$

Differentiating w.r.t. x , we get $f'(x) = 1 - \cos x$

For function to be decreasing, $f'(x) < 0$

$\Rightarrow 1 - \cos x < 0 \Rightarrow \cos x > 1$, which is not possible, because maximum value of $\cos x$ is 1.

$\therefore f(x) = (x - \sin x)$ doesn't decrease at any value of x .

14. (d) : Area of shaded region

$= \int_0^1 (y - y^2) dy = \frac{1}{6}$ sq. unit

15. (a) : Given, $f(x) = \log(1+x) - \frac{2x}{2+x}$

$\Rightarrow f'(x) = \frac{x^2}{(x+1)(x+2)^2}$

Clearly, $f'(x) > 0$ for all $x > -1$, hence $f(x)$ is increasing on $(-1, \infty)$.

16. (c)

17. (b) : We have, $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$\therefore I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx = \int_0^{\pi/4} t dt = \left[\frac{t^2}{2} \right]_0^{\pi/4} = \frac{\pi^2}{32}$

18. (b) : Given line is $\frac{x-3}{2} = \frac{y-4}{3}, z-2=0$

Vector in the direction of this line, $\vec{b} = 2\hat{i} + 3\hat{j}$

$\therefore$ Equation of line passing through the point $(-1, 3, 5)$ and parallel to $\vec{b} = 2\hat{i} + 3\hat{j}$ is given by $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$

19. (c) : Given, $|2\vec{a} - \vec{b}| = 5 \Rightarrow |2\vec{a} - \vec{b}|^2 = 5^2$
 $\Rightarrow 4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a} \cdot \vec{b} = 25 \Rightarrow 16 + 9 - 4\vec{a} \cdot \vec{b} = 25$
 $\Rightarrow \vec{a} \cdot \vec{b} = 0$

$\therefore$ Assertion is true.

Given, $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \Rightarrow |\vec{P}||\vec{Q}| \cos \theta = |\vec{P}||\vec{Q}|$

$\Rightarrow \cos \theta = 1 \Rightarrow \theta = 0^\circ$

$\therefore$ Reason is false.

20. (d) : We know that equation of family of conics whose centre lies at $(0, 0)$ is given by

$Ax^2 + 2Hxy + By^2 = 1$... (i)

which contains three unknown i.e., A, H and B

$\therefore$ Order of given differential equation is 3.

Given, $y = C_1 e^{2x+C_3} + C_3 e^{2x+C_4} C_e$

$= e^{2x} [C_1 e^{C_2} + C_3 e^{C_4}] = K e^{2x}$

$\frac{dy}{dx} = 2K e^{2x} = 2y \therefore \frac{dy}{dx} = 2y$

$\therefore$ Order is 1

$\therefore$ Assertion is false but Reason true.

21. Let $\cot^{-1} \frac{3}{4} = \theta \Rightarrow \cot \theta = \frac{3}{4}$

$\therefore \operatorname{cosec} \theta = \frac{5}{4}$

$\Rightarrow \sin \theta = \frac{4}{5} \Rightarrow \theta = \sin^{-1} \frac{4}{5}$

So, $\sin \left(\cot^{-1} \frac{3}{4} \right) = \sin \left(\sin^{-1} \frac{4}{5} \right) = \frac{4}{5}$

OR

Principal value of $\cos^{-1} \left(\frac{-1}{2} \right)$ is $\frac{2\pi}{3}$

and principal value of $\sin^{-1} \left(\frac{-1}{2} \right)$ is $\left(\frac{-\pi}{6} \right)$.

$\therefore$ Principal value of $\cos^{-1} \left(\frac{-1}{2} \right) + 2 \sin^{-1} \left(\frac{-1}{2} \right)$

$= \frac{2\pi}{3} + \left(2 \times \frac{-\pi}{6} \right) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$

22. We have, $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 500 \Rightarrow |\vec{a}|^2 |\vec{b}|^2 = 500$

$\Rightarrow |\vec{a}|^2 \times 25 = 500 \Rightarrow |\vec{a}|^2 = \frac{500}{25} = 20 \Rightarrow |\vec{a}| = 2\sqrt{5}$

23. We have, $f(x) = [x]$

$$Rf'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h} = 0$$

($\because [1+h] = 1$ and $[1] = 1$)

$$\text{and } Lf'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \infty$$

($\because [1-h] = 0$ and $[1] = 1$)

Thus $Rf'(1) \neq Lf'(1)$.

Hence, $f(x) = [x]$ is not differentiable at $x = 1$.

OR

Given, $f(0) = 2 \log 3$... (i)

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{(3^{\sin x} - 1)^2}{x \log(1+x)} \\ &= \lim_{x \rightarrow 0} \left(\frac{3^{\sin x} - 1}{\sin x} \right)^2 \cdot \left(\frac{\sin x}{x} \right)^2 \\ &= \lim_{x \rightarrow 0} \frac{(1/x) \log(1+x)}{\log(1+x)} \\ &= \lim_{x \rightarrow 0} \frac{(3^{\sin x} - 1)^2 \cdot \left(\frac{\sin x}{x} \right)^2}{\log(1+x)} = (\log 3)^2 \end{aligned}$$

... (ii)

From (i) and (ii), $\lim_{x \rightarrow 0} f(x) \neq f(0)$

$\therefore f$ is discontinuous at $x = 0$.

24. We have, $\frac{dv}{dt} = 9 \text{ cm}^3/\text{s}$ and $l = 10 \text{ cm}$

Volume of cube (v) = l^3

$$\therefore \frac{dv}{dt} = \frac{d}{dt}(l^3) = 9 \Rightarrow 3l^2 \frac{dl}{dt} = 9 \Rightarrow \frac{dl}{dt} = \frac{3}{l^2}$$

... (i)

And, surface area of cube (A) = $6l^2$

$$\therefore \frac{dA}{dt} = \frac{d}{dt}(6l^2) = 12l \frac{dl}{dt} = 12l \times \frac{3}{l^2} = \frac{36}{l}$$

(From (i))

$$\therefore \left(\frac{dA}{dt} \right)_{l=10} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{s}$$

25. Here, $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$

Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$$= \hat{i}(12+12) - \hat{j}(10+14) + \hat{k}(30-42)$$

$$= 24\hat{i} - 24\hat{j} - 12\hat{k} = 12(2\hat{i} - 2\hat{j} - \hat{k})$$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$$

26. We have, minimize $z = 5x + 7y$,

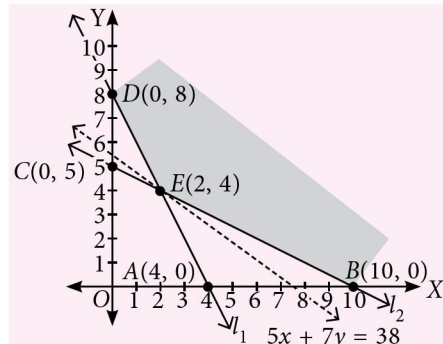
Subject to constraints, $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

To solve LPP graphically, we convert inequations into equations.

Now, $l_1 : 2x + y = 8$, $l_2 : x + 2y = 10$ and $x = 0$, $y = 0$

l_1 and l_2 intersect each other at $E(2, 4)$

Let us draw the graph of these equations as shown below.



The corner points of the feasible region are $D(0, 8)$, $B(10, 0)$ and $E(2, 4)$.

Corner points	Value of $z = 5x + 7y$
$D(0, 8)$	56
$B(10, 0)$	50
$E(2, 4)$	38 (Minimum)

From the table, we find that 38 is the minimum value of z at $E(2, 4)$. Since the region is unbounded, so we draw the graph of inequality $5x + 7y < 38$ to check whether the resulting open half plane has any point common with the feasible region. Since it has no point in common. So, the minimum value of z is obtained at $E(2, 4)$ and the minimum value of $z = 38$.

27. Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \int \frac{(x+1)+1}{\sqrt{x^2+2x+3}} dx$

Put $x^2 + 2x + 3 = t$

$$\Rightarrow (2x+2)dx = dt \Rightarrow (x+1)dx = \frac{1}{2} dt$$

$$\therefore I = \frac{1}{2} \int t^{-1/2} dt + \int \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})^2}} dx$$

$$= \frac{1}{2} \frac{t^{1/2}}{(1/2)} + \log |(x+1) + \sqrt{x^2+2x+3}| + C$$

$$= \sqrt{x^2+2x+3} + \log |(x+1) + \sqrt{x^2+2x+3}| + C$$

OR

$$\text{Let } I = \int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$$

Using partial fraction, we get

$$\begin{aligned}
 I &= -\frac{1}{7} \int \frac{1}{x^2+2^2} dx + \frac{8}{7} \int \frac{1}{x^2+5^2} dx \\
 &= -\frac{1}{7} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{7} \times \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right) + C \\
 &= -\frac{1}{14} \tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{35} \tan^{-1}\left(\frac{x}{5}\right) + C
 \end{aligned}$$

28. We have, $y + x \frac{dy}{dx} = x - y \frac{dy}{dx}$

$$\Rightarrow x \frac{dy}{dx} + y \frac{dy}{dx} = x - y \Rightarrow \frac{dy}{dx} = \frac{x-y}{x+y}$$

...(i)

This is a linear homogeneous D.E.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$$\begin{aligned}
 v + x \frac{dv}{dx} &= \frac{x - vx}{x + vx} = \frac{1-v}{1+v} \\
 \Rightarrow \frac{(1+v)}{v^2+2v-1} dv &= -\frac{dx}{x}
 \end{aligned}$$

Integrating both sides, we get

$$\begin{aligned}
 \frac{1}{2} \log |v^2+2v-1| + \log |x| &= \log C \\
 \Rightarrow y^2 + 2xy - x^2 &= \pm C^2 \\
 \Rightarrow y^2 + 2xy - x^2 &= C_1 \text{ (where } C_1 = \pm C^2)
 \end{aligned}$$

OR

We have, $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$

$$\Rightarrow x e^x dx + \frac{y}{\sqrt{1-y^2}} dy = 0$$

Integrating both sides, we get

$$\begin{aligned}
 x \cdot e^x - \int 1 \cdot e^x dx - \frac{1}{2} \int (1-y^2)^{-\frac{1}{2}} (-2y) dy &= C \\
 \Rightarrow x e^x - e^x - \frac{1}{2} \frac{(1-y^2)^{\frac{1}{2}}}{1/2} &= C \\
 \Rightarrow e^x (x-1) - \sqrt{1-y^2} &= C
 \end{aligned}$$

29. Let E_1 be the event that the outcome on the die is 1 or 2, E_2 be the event that the outcome on the die is 3, 4, 5, 6.

$$\therefore P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let A be the event of getting exactly one tail.

Now, $P(A/E_1)$ be the probability of getting exactly one tail by tossing the coin three times if she gets 1 or 2 = $3/8$ and $P(A/E_2)$ be the probability of getting exactly one tail in a single throw of coin if she gets 3, 4, 5, 6 = $1/2$. The probability that the girl threw 3, 4, 5, 6 with the die, if she obtained exactly one tail is given by $P(E_2/A)$.

$$\begin{aligned}
 \therefore P(E_2/A) &= \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \\
 &= \frac{\frac{2}{6} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{3}{8} + \frac{2}{6} \cdot \frac{1}{2}} = \frac{8}{11}
 \end{aligned}$$

OR

Let p_1 be the event that A speaks truth and p_2 be the event that B speaks truth, then A and B are independent events such that

$$\begin{aligned}
 p_1 &= P(A \text{ speaks the truth}) = 60\% = \frac{60}{100} = \frac{6}{10} \\
 \therefore q_1 &= P(A \text{ does not speak truth i.e., tells a lie}) \\
 &= 1 - \frac{6}{10} = \frac{4}{10}
 \end{aligned}$$

$$p_2 = P(B \text{ speaks the truth}) = 90\% = \frac{90}{100} = \frac{9}{10}$$

$$\therefore q_2 = P(B \text{ tells a lie}) = 1 - \frac{9}{10} = \frac{1}{10}$$

$$\begin{aligned}
 p(\text{contradiction}) &= p_1 q_2 + q_1 p_2 \\
 &= \frac{6}{10} \times \frac{1}{10} + \frac{4}{10} \times \frac{9}{10} = \frac{6+36}{100} = \frac{42}{100} = 0.42
 \end{aligned}$$

$$\therefore \text{Required \%} = \frac{42}{100} \times 100 = 42\%$$

Hence, A and B are likely to contradict each other in 42% of the cases.

30. Let $I = \int_0^{\pi/4} \frac{\sin 2\theta}{\sin^4 \theta + \cos^4 \theta} d\theta$

$$= \int_0^{\pi/4} \frac{2 \sin \theta \cos \theta}{\sin^4 \theta + (1 - \sin^2 \theta)^2} d\theta$$

Put $\sin^2 \theta = t \Rightarrow 2 \sin \theta \cos \theta d\theta = dt$

Also, $\theta = 0 \Rightarrow t = 0$ and $\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{2}$

$$\begin{aligned}
 \therefore I &= \int_0^{1/2} \frac{dt}{t^2 + (1-t)^2} = \int_0^{1/2} \frac{dt}{2t^2 - 2t + 1} \\
 &= \frac{1}{2} \int_0^{1/2} \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}
 \end{aligned}$$

$$= \left[\tan^{-1} \left[\frac{2t-1}{1} \right] \right]_{t=0}^{t=1} = \tan^{-1} 0 - \tan^{-1}(-1) = \frac{\pi}{4}$$

31. The given lines are parallel.

Here, $\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{a}_2 = 2\hat{i} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} - 3\hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{j} - 3\hat{k}$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \hat{i}(6+3) - \hat{j}(-3-3) + \hat{k}(-1+2) \\ = 9\hat{i} + 6\hat{j} + \hat{k}$$

$$|\vec{b}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\text{Shortest distance} = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{\sqrt{59}}{\sqrt{7}} \text{ units}$$

32. We have, $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$$\Rightarrow BA = 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow BA = 4I$$

$$\Rightarrow B^{-1}(BA) = 4B^{-1}I \quad [\text{Pre multiplying by } B^{-1}]$$

$$\Rightarrow 4B^{-1} = IA \Rightarrow B^{-1} = \frac{1}{4}A \quad \dots(i)$$

Now, given system of equations can be written as $BX = C$, where

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\text{or } X = B^{-1}C \Rightarrow X = \frac{1}{4}AC \quad [\text{Using (i)}]$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \therefore x = 2, y = 1, z = -1.$$

33. Given, $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0) = \frac{x}{1-x}$
 $(\because x \in (-\infty, 0), |x| = -x)$

For one-one : Let $f(x_1) = f(x_2), x_1, x_2 \in (-\infty, 0)$

$$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1(1-x_2) = x_2(1-x_1)$$

$$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore f$ is one-one

For onto : Let $f(x) = y$

$$\Rightarrow y = \frac{x}{1-x} \Rightarrow y(1-x) = x \Rightarrow y - xy = x$$

$$\Rightarrow x + xy = y \Rightarrow x(1+y) = y \Rightarrow x = \frac{y}{1+y}$$

Here, $y \in (-1, 0)$

So, x is defined for all values of y .

Also $x \in (-\infty, 0)$ for all $y \in (-1, 0)$.

$\therefore f$ is onto.

OR

Refer to answer 19, Page no. 16, Class 12, MTG 100 percent Mathematics.

34. The given line is

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda (\text{say}) \quad \dots(i)$$

Any point on it is

$$M(10\lambda + 11, -4\lambda - 2, -11\lambda - 8).$$

Let the given point be $P(2, -1, 5)$

Dr's of PM are $10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5$.

i.e., $10\lambda + 9, -4\lambda - 1, -11\lambda - 13$.

As line (i) is $\perp$ to PM , so

$$(10\lambda + 9) \cdot 10 + (-4\lambda - 1) \cdot (-4) + (-11\lambda - 13) \cdot (-11) = 0$$

$$\Rightarrow (100 + 16 + 121)\lambda + (90 + 4 + 143) = 0$$

$$\Rightarrow 237\lambda + 237 = 0 \Rightarrow \lambda = -1.$$

$\therefore$ Dr's of PM are $-1, 3, -2$ i.e., $1, -3, 2$.

$$\therefore \text{Eq. of the line } PM \text{ is } \frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-5}{2} \quad \dots(ii)$$

If $Q(\alpha, \beta, \gamma)$ is the image of P in line (i), then M is the mid point of PQ .

Here, $M \equiv (1, 2, 3)$

$$\therefore \frac{\alpha+2}{2} = 1, \frac{\beta-1}{2} = 2, \frac{\gamma+5}{2} = 3 \Rightarrow \alpha = 0, \beta = 5, \gamma = 1$$

$\therefore Q(0, 5, 1)$ is the image of P in line (i).

$$\text{and } PQ = \sqrt{(2-0)^2 + (-1-5)^2 + (5-1)^2} = 2\sqrt{14} \text{ units}$$

OR

Refer to answer 34, Page no. 498, Class 12, MTG 100 percent Mathematics.



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35. The given lines are

$$2x + y = 4$$

$$3x - 2y = 6$$

$$x - 3y + 5 = 0$$

From (i) and (ii), we get

$$4 - 2x = \frac{3x - 6}{2} \Rightarrow x = 2$$

From (i), we get $y = 4 - 2x = 4 - 2 \times 2 = 0$

$\therefore$ Lines (i) and (ii) intersect each other at the point (2, 0)

Similarly, lines (ii) and (iii) intersect each other at the point (4, 3).

Also, lines (i) and (iii) intersect each other at the point (1, 2).

$\therefore$ Required area

$$\begin{aligned} &= \int_1^4 \left(\frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left(\frac{3x-6}{2} \right) dx \\ &= \frac{1}{3} \left[\frac{(x+5)^2}{2} \right]_1^4 - [4x - x^2]_1^2 - \frac{1}{2} \left[\frac{3x^2}{2} - 6x \right]_2^4 \\ &= \frac{7}{2} \text{ sq. units} \end{aligned}$$

36. (i) $P(X = 2) = (\text{Probability of not getting six at first throw}) \times (\text{Probability of getting six at second throw})$

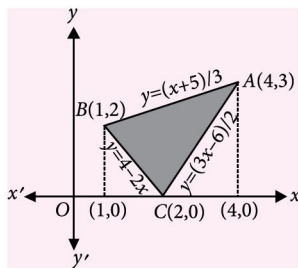
$$= \frac{5}{6} \times \frac{1}{6} = \frac{5}{6^2}$$

$$(ii) P(X = 4) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5^3}{6^4}$$

$$\begin{aligned} (iii) P(X \geq 6) &= \left(\frac{5}{6} \right)^5 \times \frac{1}{6} + \left(\frac{5}{6} \right)^6 \times \frac{1}{6} + \dots \infty \\ &= \frac{5^5}{6^6} \left[1 + \frac{5}{6} + \left(\frac{5}{6} \right)^2 + \dots \infty \right] = \frac{5^5}{6^6} \left[\frac{1}{1 - \frac{5}{6}} \right] = \left(\frac{5}{6} \right)^5 \end{aligned}$$

OR

$$\begin{aligned} P(X \geq 4) &= \left(\frac{5}{6} \right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6} \right)^4 \cdot \frac{1}{6} + \dots \\ &= \frac{5^3}{6^4} \left[1 + \left(\frac{5}{6} \right) + \left(\frac{5}{6} \right)^2 + \dots \right] \\ &= \frac{5^3}{6^4} \left[\frac{1}{1 - \left(\frac{5}{6} \right)} \right] = \left(\frac{5^3}{6^4} \right) \cdot 6 = \frac{5^3}{6^3} \end{aligned}$$



37. (i) Let x be the number of extra days after 1st August.

$$\therefore \text{Price} = ₹(600 - 3 \times x) = ₹(600 - 3x)$$

$$\begin{aligned} \text{Quantity} &= 80 \text{ quintals} + x(1 \text{ quintal per day}) \\ &= (80 + x) \text{ quintals} \end{aligned}$$

(ii) $R(x) = \text{Quantity} \times \text{Price}$

$$\begin{aligned} &= (80 + x)(600 - 3x) = 48000 - 240x + 600x - 3x^2 \\ &= 48000 + 360x - 3x^2 \end{aligned}$$

(iii) We have, $R(x) = 48000 + 360x - 3x^2$

$$\Rightarrow R'(x) = 360 - 6x \Rightarrow R''(x) = -6$$

For $R(x)$ to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$$\Rightarrow 360 - 6x = 0 \Rightarrow x = 60$$

After 60 days from 1st August Shyam's father attain maximum revenue.

OR

Maximum revenue is collected by Shyam's father when $x = 60$.

$$\therefore \text{Maximum revenue} = R(60)$$

$$= 48000 + 360(60) - 3(60)^2 = 48000 + 21600 - 10800 = 58800$$

38. (i) Given, height of open box = x cm

Length of open box = $20 - 2x$

and width of open box = $20 - 2x$

$\therefore$ Volume (V) of the open box

$$= x \times (20 - 2x) \times (20 - 2x)$$

(ii) We have, $V = x(20 - 2x)^2$

$$\therefore \frac{dV}{dx} = x \cdot 2(20 - 2x)(-2) + (20 - 2x)^2$$

$$= (20 - 2x)(-4x + 20 - 2x) = (20 - 2x)(20 - 6x)$$

$$\text{Now, } \frac{dV}{dx} = 0 \Rightarrow 20 - 2x = 0 \text{ or } 20 - 6x = 0$$

$$\Rightarrow x = 10 \text{ or } \frac{10}{3}$$

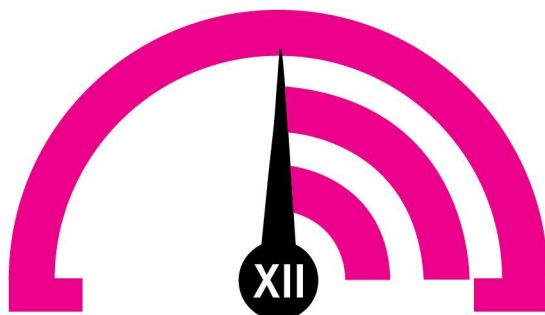
$$\therefore \text{Height of box} = \frac{10}{3} \quad [\text{As, } x = 10 \text{ is not possible}]$$



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- | | |
|---------------------|-----------------|
| 1. {3, 6} | 11. 12 |
| 2. 207! | 12. 4 |
| 3. 0.25 | 13. $\pi/3$ |
| 4. $[1/2, \infty)$ | 14. $1/x$ |
| 5. $(-\infty, 1]$ | 15. $[-2, -1]$ |
| 6. $-4i$ | 16. $-1/3$ |
| 7. ${}^{10}C_5 x^5$ | 17. 1 |
| 8. n | 18. null |
| 9. 1 | 19. Rectangular |
| 10. 20 | 20. $2\pi/3$ |

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Total Marks : 80

Series 7 : Integrals and Application of Integrals

Time Taken : 60 Min.

Only One Option Correct Type

- The value of $\int_0^1 \frac{e^x(1+x)\sin^2(xe^x)dx}{\sin^2(xe^x)+\sin^2(e-xe^x)}$ is
(a) $\frac{e}{4}$ (b) $\frac{e}{2}$ (c) $2e$ (d) $\frac{e}{8}$
- If $f'''(x) = k$ in $[0, a]$, then $\int_0^a f(x)dx - \left\{ xf(x) - \frac{x^2}{2!}f'(x) + \frac{x^3}{3!}f''(x) \right\}_0^a$ is
(a) $-ka^4/12$ (b) $ka^4/24$
(c) $-ka^4/24$ (d) None of these
- If A_n is the area bounded by $y = x$ and $y = x^n$, $n \in \mathbb{N}$, then $A_2 \cdot A_3 \dots A_n =$
(a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{2^n n(n+1)}$
(c) $\frac{1}{2^{n-1}n(n+1)}$ (d) $\frac{1}{2^{n-2}n(n+1)}$
- $\int \frac{x^2 dx}{(a+bx)^2} = A(a+bx) - \frac{B}{a+bx} + C \ln(a+bx) + D$, then $A+B+C=$
(a) $\frac{1+2a-a^2}{b^3}$ (b) $\frac{(1-a)^2}{b^3}$
(c) $\frac{1-2a-a^2}{b^3}$ (d) $\frac{(1+a)^2}{b^3}$
- If $\int \frac{x^2}{\sqrt{1-x}} dx = A(1-x)^{1/2} + B(1-x)^{3/2} + C(1-x)^{5/2} + D$, then $A+B+C=$

(a) $-\frac{8}{15}$ (b) $-\frac{16}{15}$ (c) $-\frac{14}{15}$ (d) $\frac{1}{15}$

- $\int_1^{16} \tan^{-1} \sqrt{\sqrt{x}-1} dx =$
(a) $\frac{8\pi}{3} + \sqrt{3}$ (b) $\frac{8\pi}{3} - \sqrt{3}$
(c) $\frac{16\pi}{3} - 2\sqrt{3}$ (d) $\frac{16\pi}{3} + 2\sqrt{3}$

One or More Than One Option(s) Correct Type

- The integral $\int_0^\pi xf(\sin x) dx$ is equal to
(a) $\frac{\pi}{2} \int_0^\pi f(\sin x) dx$ (b) $\frac{\pi}{4} \int_0^\pi f(\sin x) dx$
(c) $\pi \int_0^{\pi/2} f(\sin x) dx$ (d) $\pi \int_0^{\pi/2} f(\cos x) dx$
- If $I_n = \int_0^{\pi/4} \tan^n x dx$, then
(a) $I_7 + I_5 = \frac{1}{6}$ (b) $I_{10} + I_8 = \frac{1}{9}$
(c) $I_8 - I_{12} = \frac{2}{99}$ (d) $I_{12} + 2I_{10} + I_8 = \frac{20}{99}$
- For which of the following values of m , is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ is equal to $9/2$ sq. units?
(a) -4 (b) -2 (c) 2 (d) 4
- If $I_n = \int_{-\pi}^\pi \frac{\sin nx}{(1+\pi^x) \sin x} dx$, $n = 0, 1, 2, \dots$, then

$$(a) I_n = I_{n+2} \quad (b) \sum_{m=0}^{10} I_{2m+1} = 10\pi$$

$$(c) \sum_{m=0}^{10} I_{2m} = 0 \quad (d) I_n = I_{n+1}$$

11. Area enclosed by parabola $ay = 3(a^2 - x^2)$ and x -axis is 64 sq. units, then value of a is
 (a) 4 (b) 2 (c) -4 (d) -2

12. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A} \log_e \left(\frac{f(x)}{g(x)} \right) + c$, then
 (a) $A = \sin(a-b)$ (b) $g(x) = \sec(x-a)$
 (c) $f(x) = \sec(x-b)$ (d) $A = \sin(b-a)$

13. $\int \frac{dx}{x^2(x^4+1)^{3/4}} = A \left(\frac{x^4+1}{x^4} \right)^B + c$, then
 (a) $A = -1$ (b) $B = \frac{1}{4}$ (c) $A = \frac{1}{2}$ (d) $B = \frac{1}{2}$

Comprehension Type

Paragraph for Q. No. 14 and 15

Consider the ellipse $E, \frac{x^2}{4} + y^2 = 1$. Let PM be the ordinate of P , a point on E in the first quadrant. Let Q be a point on PM such that QM is equal to the length of the subtangent at P . Let Q trace the curve C as P varies.

14. If the curve C intersects E at the points A and B , the area of triangle OAB is

$$(a) \sqrt{5} \quad (b) \frac{1}{\sqrt{5}} \quad (c) \frac{2}{\sqrt{5}} \quad (d) \frac{\sqrt{5}}{2}$$

15. The area enclosed by C and E is

$$(a) \cos^{-1} \frac{2}{\sqrt{5}} + 2 \ln \frac{5}{4} \quad (b) \sin^{-1} \frac{2}{\sqrt{5}} + 2 \ln \frac{5}{4}$$

$$(c) \sin^{-1} \frac{2}{\sqrt{5}} - 2 \ln \frac{5}{4} \quad (d) \cos^{-1} \frac{2}{\sqrt{5}} - 2 \ln \frac{5}{4}$$

Matrix Match Type

16. If the value of $\lim_{n \rightarrow \infty} f(n)$, is given in Column-II and $f(n)$ is given in Column-I, then match the following.

Column-I		Column-II	
(P)	$\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2-1}} + \dots + \frac{1}{\sqrt{n^2-(n-1)^2}} \right)$	(1)	$\frac{\pi}{4} + \frac{1}{2} \log 2$
(Q)	$\left(\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \dots + \frac{1}{n} \right)$	(2)	$1/2$
(R)	$\frac{1}{n} \left[\sin^2 \frac{\pi}{2n} + \sin^2 \frac{2\pi}{2n} + \dots + \sin^2 \frac{n\pi}{2n} \right]$	(3)	$\pi/6$
(S)	$\left(\frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-2^2}} + \dots + \frac{1}{\sqrt{4n^2-n^2}} \right)$	(4)	$\pi/2$

P	Q	R	S
(a) 4	1	2	3
(b) 2	1	3	4
(c) 1	2	3	4
(d) 4	2	1	3

Numerical Answer Type

17. If $\int_0^1 \frac{dx}{(1+x)(2+x)\sqrt{x(1-x)}} = \frac{\pi A}{\sqrt{6}(\sqrt{3}+1)(157)}$, then A is equal to _____.
18. If the area bounded by the curves $x^2 = y, x^2 = -y$ and $y^2 = 4x - 3$ is $\frac{m}{n}$, where m and n are relatively prime positive integers, then $m + n$ is _____.
19. If A is the area of the greater region bounded by the curve $y = \cos x, y = x + 1$ and $y = 0$ in sq. units then value of $6A$ is _____.
20. A biquadratic function $y = ax^4 + bx^3 + cx^2 + dx + e$ ($a \neq 0$) touches $y = px + q$ at $x = \alpha, \beta$ ($\alpha < \beta$). Area of the region bounded by these graphs is $\frac{a(\beta-\alpha)^5}{\lambda}$, then $\frac{\lambda}{5} =$ _____.



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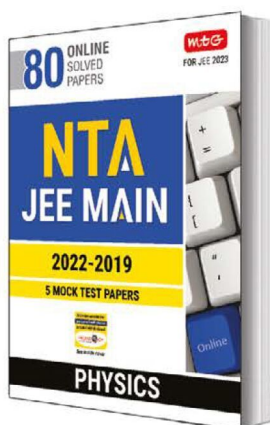
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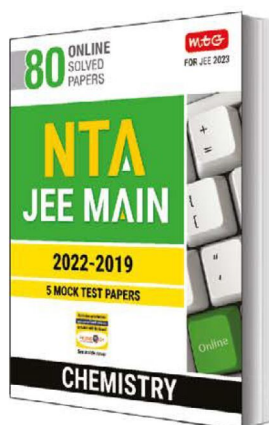


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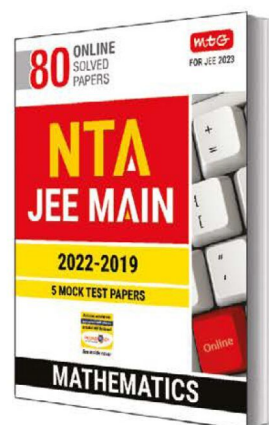
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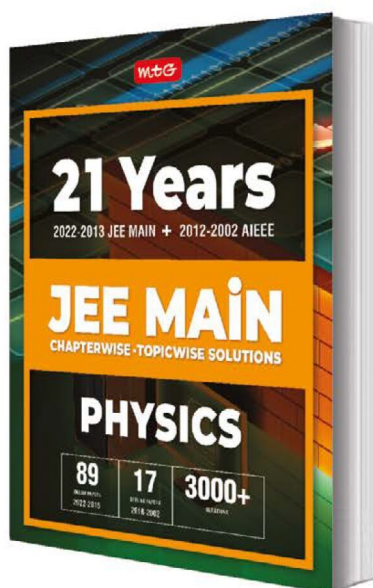
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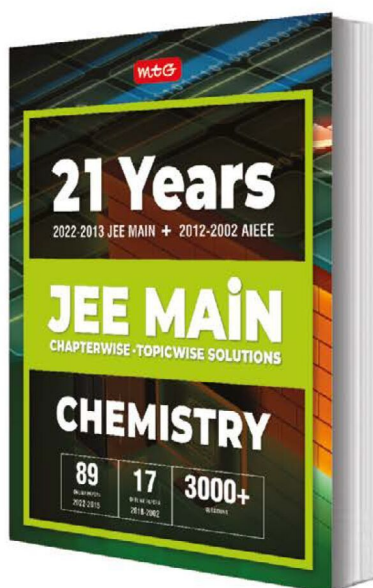


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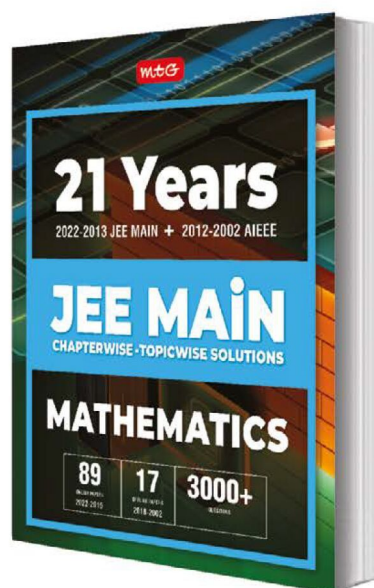
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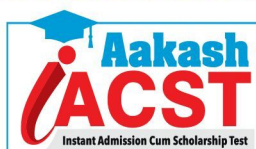
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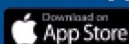
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